THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXI — JANUARY 1905 — NUMBER 1

RADIATION THROUGH A FOGGY ATMOSPHERE

By ARTHUR SCHUSTER

1. In discussing the transmission of light through a mass of gas, it is usual to consider only the effects of emission and absorption, and to neglect all effects of scattering. But when the absorbing mass holds fine particles of matter in suspension, the scattered light materially affects the character of the transmitted radiation. I propose to discuss the conditions under which "bright line" spectra or "dark line" spectra may be obtained from a radiating mass of gas, taking account of scattering. I call an atmosphere "foggy" when scattering takes place to an appreciable extent. The applications of the results of this investigation are, however, much wider than the title chosen would seem to imply, for there is some scattering even from the molecules of a homogeneous substance, and to that extent all bodies fall within the definition and may be called "foggy."

According to the investigations of Lord Rayleigh, the greater part of the light we receive from the sky is due to light scattered by the molecules of the air. This involves a diminution in the intensity of the direct rays amounting in our atmosphere to roughly 5 per cent. The effective thickness of stellar atmospheres may be great compared with that of the shell of air which surrounds our globe, and hence the effects of scattering may be of primary importance in interpreting the nature of stellar atmospheres.

2. The following notation will be used:

E = the total energy of radiation within a certain small range of wave-lengths sent out by unit surface of a completely black surface. E is a function of the temperature and wave-length.

S = the total energy of radiation incident within the same limit of wave-lengths on unit surface of a plane layer of the foggy gas.

R = the energy which leaves the plane layer per unit surface.

R_o = the particular value of R for the case that there is no absorption.

R_c = the particular value of R for rays which are completely absorbed by an infinitely thin layer.

κ = the coefficient of absorption.

s = the coefficient of scattering.

The object of the investigation is to determine R in terms of S and E, if E refers to the temperature of the foggy gas. It will save needless repetition if it is understood once for all that our statements always refer to unit surface of the radiating or absorbing layer.

κ is a function of the wave-length which also depends on the density of the medium. If the medium is uniform, all molecules absorbing alike, κ would be proportional to the density. But in a mixture of different gases, κ must be considered proportional only to the quantity (measured per unit volume) of the particular substance which absorbs the wave-length in question. Similarly s depends on the number of the scattering particles, the scattering and absorbing particles not necessarily being of the same nature. If the scattering is of the nature of that which causes the blue color of the sky, the value of s varies inversely as the fourth power of λ, but in case of an ordinary cloud or mist, the dependence on wave-length is much less marked.

If S be the total intensity of radiation incident on a layer of small thickness dx, the radiation absorbed by the layer is $\kappa S dx$. The light emitted by the same layer in each direction is thus $\kappa E dx$. This follows from the law connecting absorption and radiation.

The light scattered by the layer is $sSdx$, of which one-half is sent forward and one-half returned backward.

The following variables will be introduced for convenience of expression:

$$\beta = \kappa / s \ ,$$

$$a = \sqrt{\kappa/(\kappa+s)} = \sqrt{\beta/(1+\beta)} \ ,$$

$$\therefore \beta = a^2/(1-a^2) \ .$$

β varies from zero to infinity, but α must lie between zero and unity.

$$\gamma=(1+\alpha)/(1-\alpha) \ ,$$

$$\therefore \gamma=1+2\beta+2\sqrt{\beta+\beta^2} \ .$$

3. Let (Fig. 1) S_1S_2 be a surface sending out the radiation S, and let this radiation after passing through part of the foggy atmosphere be reduced to a value A and fall on a thin layer of thickness dx. The effect of the layer is to absorb energy amounting to κAdx, and additionally to reduce the incident light by a quantity $sAdx$, which is not absorbed, but sent in equal amounts backward and forward as scattered light. If the stream of radiant energy in the opposite direction is B, we have similarly a diminution of energy equal to $(\kappa+s)\ B$, of which, however, $\frac{1}{2}sB$ is sent both forward and backward as scattered light. The layer also radiates energy in both directions equal to κEdx. Collecting these effects, we obtain the equations:

FIG. 1.

$$\frac{dA}{dx}=\kappa(E-A)+\tfrac{1}{2}s(B-A) \qquad (1)$$

$$\frac{dB}{dx}=\kappa(B-E)+\tfrac{1}{2}s(B-A) \ . \qquad (2)$$

Combining (1) and (2) we find:

$$\frac{d(A+B)}{dx}=(\kappa+s)(B-A) \qquad (3)$$

$$\frac{d(A-B)}{dx}=2\kappa E-\kappa(A+B) \ . \qquad (4)$$

Differentiating (3) and with the help of (4)

$$\frac{d^2(A+B)}{dx^2}=\kappa(\kappa+s)(A+B-2E) \ . \qquad (5)$$

If E is constant or varies uniformly with x, the last equation may be integrated, and we derive:

$$(A+B-2E)=Ke^{(\kappa+s)\alpha x}+K_1e^{-(\kappa+s)\alpha x} \qquad (6)$$

where K and K_1 are two constants and α has the value assigned to it in § 2.

If the temperature of the medium is constant, so that E has the same value throughout the scattering medium, differentiation gives

$$\frac{d(A+B)}{dx}=a(\kappa+s)(Ke^{(\kappa+s)ax}-K_1e^{-(\kappa+s)ax}) \tag{7}$$

and hence by introducing (3)

$$B-A=a(Ke^{(\kappa+s)ax}-K_1e^{-(\kappa+s)ax}) \ . \tag{8}$$

Equations (6) and (8) now allow us to obtain A and B separately, and we thus find:

$$\left.\begin{aligned} 2A&=2E+(1-a)Ke^{(\kappa+s)ax}+(1+a)K_1e^{-(\kappa+s)ax} \\ 2B&=2E+(1+a)Ke^{(\kappa+s)ax}+(1-a)K_1e^{-(\kappa+s)ax} \end{aligned}\right\} \ . \tag{9}$$

We consider x to be measured from the front surface of the foggy medium in the direction in which the radiation A proceeds. If no radiation enters the medium from the opposite direction, and if the radiation incident in the first absorbing layer be S, we have the conditions:

$$\left.\begin{aligned} &\text{for } x=0; \qquad B=0 \\ &\text{for } x=-t; \qquad A=S \end{aligned}\right\} \tag{10}$$

the thickness of the medium being denoted by t.

We require to determine the emergent radiation which is equal to the value which A acquires when $x=0$. Denoting this by R, we have from the first of equations (9)

$$2R=2E+(1-a)K+(1+a)K_1 \ . \tag{11}$$

Introducing (10) into (9) allows us to determine K and K_1. We obtain in the first place the equations

$$0=2E+(1+a)K+(1-a)K_1$$

$$2S=2E+(1-a)Ke^{-a(\kappa+s)t}+(1+a)K_1e^{a(\kappa+s)t} \ ,$$

and these give

$$K=\frac{2\left[(1-a)-(1+a)e^{a(\kappa+s)t}\right]E-2(1-a)S}{(1+a)^2e^{a(\kappa+s)t}-(1-a)^2e^{-a(\kappa+s)t}} \ ,$$

$$K_1=\frac{2\left[(1-a)e^{-a(\kappa+s)t}-(1+a)\right]E+2(1+a)S}{(1+a)^2e^{a(\kappa+s)t}-(1-a)^2e^{-a(\kappa+s)t}} \ .$$

Finally by substitution into (11)

$$R=2a\frac{\left[(1+a)e^{a(\kappa+s)t}+(1-a)e^{-a(\kappa+s)t}\right]E+2(S-E)}{(1+a)^2e^{a(\kappa+s)t}-(1-a)^2e^{-a(\kappa+s)t}} \ . \tag{12}$$

Equation (12) contains the solution of our problem.

4. The equations of the last paragraph have been deduced under the assumption that the radiation throughout the absorbing mass is uniformly distributed in such a way that it does not depend on the angle between any direction considered and the normal drawn toward the same side. This supposition is obviously incorrect, for it appears that, even if it were to hold at any surface, e. g., the first surface of the layer dx (Fig. 1), absorption in that layer would destroy the uniformity owing to the greater absorption which the oblique rays suffer. To some extent the effect of scattering would act in the sense of partly restoring the equality of distribution; nevertheless serious errors might be introduced, if we attempted to obtain accurate values of κ and s by means of the application of equation (12). The complete investigation leads to equations of such complexity that a discussion becomes impossible, and I shall only use the solution obtained under the simplified conditions to deduce certain consequences which cannot be affected by the assumption made. The error committed might be allowed for by taking s and κ to be functions of the distance. When considered in this light, it is seen how useless the more complete calculation would be, because in the more important cases to which we have to apply our results, the coefficients of scattering and absorption vary in an unknown manner, and the error committed by the simplification of this problem becomes merged in other unavoidable uncertainties.

5. Before discussing the general results contained in equation (12) we may treat separately of some simple special cases. When the coefficient of absorption, and consequently a, is zero, we require to express the exponentials of (12) in a series, the first two terms being retained. But it is easier in this case to proceed directly. Equations (3) and (4) in this case become

$$\frac{d(A+B)}{dx}=s(B-A) \ ,$$

$$\frac{d(A-B)}{dx}=0 \ .$$

The second equation shows that $A-B$ is a constant which must be equal to R_0, the value of A at the front surface. The first equation may now be integrated, and gives

$$A+B=a-sR_0x \ .$$

As for $x=0$, $B=0$, and $A=R_0$, it follows that $a=R_0$; or replacing B by $A-R$,

$$2(A-R_0)=-sR_0x\ .$$

When $x=-t$, the value of A is equal to S, the incident radiation, hence

$$2(S-R_0)=sA_0t\ ;$$

or finally:

$$R_0=\frac{2}{2+st}S\ . \tag{13}$$

The equation shows that the emergent radiation diminishes with increasing thickness, but not so quickly as it would do if scattering acted in the same manner as absorption. If, for instance, we give to st the numerical value of ninety-eight, so that the emergent light is 2 per cent. of the incident light, doubling the layer would still give us 1 per cent. for the transmitted light, and with greater thicknesses the light would, roughly speaking, be inversely proportional to the thickness. But in the case of absorption, the double layer would only transmit 2 per cent. of 2 per cent., and the transmitted light would diminish in a geometric ratio, while the thickness increases in an arithmetic ratio.

6. When either st or κt is so large that practically no part of the original light is transmitted, we may neglect in (12) all terms except those containing an exponential with a positive argument, and this gives at once

$$R_c=\frac{2a}{1+a}E \tag{14}$$

When κ is large compared with s, a approaches unity and ultimately $R_c=E$. The radiation in that case becomes equal to that of a completely black surface, which agrees with the well-known law that absorption irrespective of scattering tends to make the radiation of all bodies equal to that of a black body when the thickness is increased.

But, as has been mentioned, scattering always exists, and has to be taken into account. It appears from the definition of a that it is always a fraction, and hence the factor of E in (14) is always smaller than one. It follows that the emergent radiation increases with the value of κ, and hence a luminous gas always gives a spec-

trum of bright lines, and does not approach with increasing thickness to the radiation from a black body, as it would do in the absence of scattering.

7. It is not possible to discuss equation (12) in its general form. In order to draw the appropriate conclusions in certain typical cases, we introduce other variables.

Put

$$e^{st}=r\ ;\qquad \beta=\kappa/s\ ,$$

and introduce a quantity γ defined by

$$\gamma=\frac{1+a}{1-a}\ .$$

We have then

$$a(\kappa+s)t=st(1+\beta)a\ ;$$

and also

$$a=\sqrt{\beta/(1+\beta)}\ .$$

Hence

$$\gamma=1+2\beta+2\sqrt{\beta+\beta^2} \tag{15}$$

$$\frac{2a}{1-a}=\gamma-1\ ,$$

$$\frac{2a(1+a)}{(1-a)^2}=\frac{1+a}{1-a}\cdot\frac{2a}{1-a}=\gamma(\gamma-1)\ ,$$

$$\frac{4a}{(1-a)^2}=\gamma^2-1\ .$$

Equation (12) now becomes

$$R=(\gamma-1)\frac{(\gamma r^{\sqrt{\beta+\beta^2}}+r^{-\sqrt{\beta+\beta^2}})E-(\gamma+1)(E-S)}{\gamma^2 r^{\sqrt{\beta+\beta^2}}-r^{-\sqrt{\beta+\beta^2}}}\cdot \tag{16}$$

As (15) gives γ in terms of β, all factors of E and S are now expressed in terms of β and st. I have carried out the calculations for the three cases that st is equal to $\frac{1}{2}$, 1, and 2, respectively, and for a number of different values of β. If we calculate the coefficient in (16) and write it (16) in the form

$$R=aE+bS\ ,$$

Table I gives the coefficients of a and b.

TABLE I

		0.1	0.8	1	1.2	2	10
$st=0.5$	a	0.0486	0.3258	0.3872	0.4428	0.6165	0.9762
	b	0.7604	0.5333	0.4820	0.4355	0.2911	0
$st=1$	a	0.0944	0.5276	0.6019	0.6626	0.8142	0.9762
	b	0.6000	0.2902	0.2364	0.1926	0.0855	0
$st=2$......	a	0.1760	0.7147	0.7716	0.8111	0.8895	0.9762
	b	0.3971	0.0872	0.0574	0.0379	0.0074	0

The relative intensities of the dark and bright lines as they would appear in the special cases considered will be most easily under-

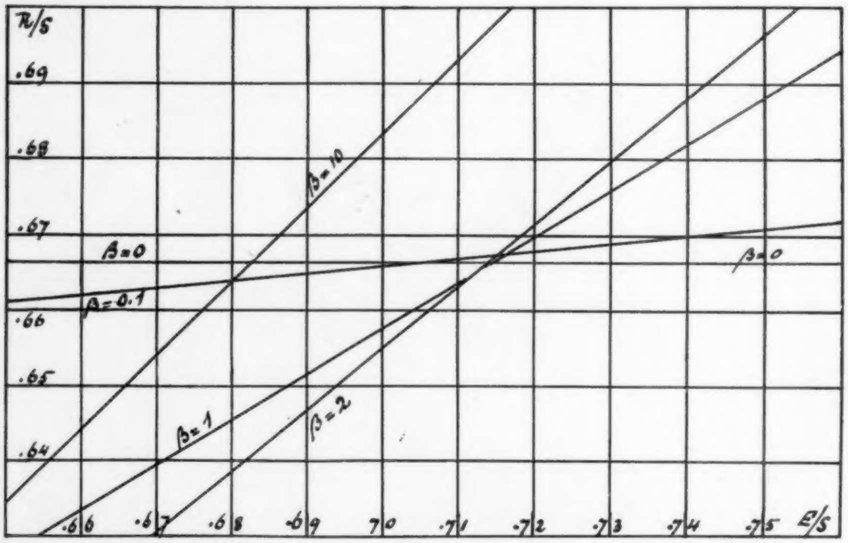


FIG. 2. $st=1$.

stood if straight lines are drawn (Fig. 2) with E/S as abscissæ and R/S as ordinates. That figure refers to the case $st=1$. The horizontal line marked $\beta=0$ gives R_0/S, which defines the intensity of the transmitted light when there is no absorption. The value of R_0/S is obtained from (13), which shows that for $st=1$, only two-thirds of the incident light traverses the scattering medium. If this medium is capable of sending out any vibration defined as regards radiative power by the fraction $\beta=\kappa/s$, the corresponding intensity may be

obtained from the curve by taking on the horizontal axis the magnitude E/S, which is the intensity of the black radiation at the temperature of the absorbing and scattering medium in terms of that of the incident light. The corresponding ordinate gives the transmitted light in terms of the same unit. If the point of the straight line corresponding to any particular value of β lies above the horizontal line marked $\beta = 0$, the appearance will be that of a bright line; if it lies below, a dark line would be observed. Starting with a comparatively low temperature of the foggy medium and gradually increasing it (i. e., gradually increasing the ordinates), it is seen that at first all homogeneous vibrations appear as dark lines, and if the temperature is sufficiently low (not shown in figure) the highest values of β give the greatest deficiency of light. This is in accordance with what takes place in the absence of scattering.

When the temperature is gradually raised, the most intense line represented in the figure ($\beta = 10$) ceases to be the darkest line, and ultimately when E/S is about 0.682, this line becomes brighter than the background. The next line to change from darkness to brightness is the line of lowest intensity $\beta = 0.1$, and when E/S is more than 0.715, all the lines are bright. The change from dark to bright lines takes place within a comparatively small range of temperature; nevertheless the possibility of the simultaneous appearance of dark and bright lines according to the intensity of absorption is shown by the figure. If, instead of a homogeneous line, we contemplate the case of narrow bands such as frequently occur, we must consider β to have a maximum value at the center of the band (e. g., $\beta = 10$) and to fall off on either side more or less rapidly to zero. At very low temperatures of the medium, the center of the band in this case would be darkest and at high temperatures brightest. But intermediate temperatures would give the appearance of a bright central line on a dark absorption band. Thus at a temperature of 0.69, the brightness of the center ($\beta = 10$) in terms of the intensity of the transmitted light is 0.674, which means that it is 1.2 per cent. brighter than the background, while towards both sides, where β has fallen to 2, the intensity is 0.637 or 4.5 per cent. darker than the background. The appearance is therefore that of an absorption band with a reversed line in the center.

$\frac{R-R_0}{R_0}$

Fig. 3 gives the diagram of radiation for $st=0.5$. It is drawn to a somewhat larger scale than Fig. 2.

Fig. 4 represents the connection between transmitted light and coefficient of absorption when $st=2$. In this case the unabsorbed radiation is scattered to the extent that only half the incident light is transmitted. The possibility of the simultaneous appearance of dark and bright lines, which carries with it the possibility of an absorption band with a reversed line at the center, is increased in

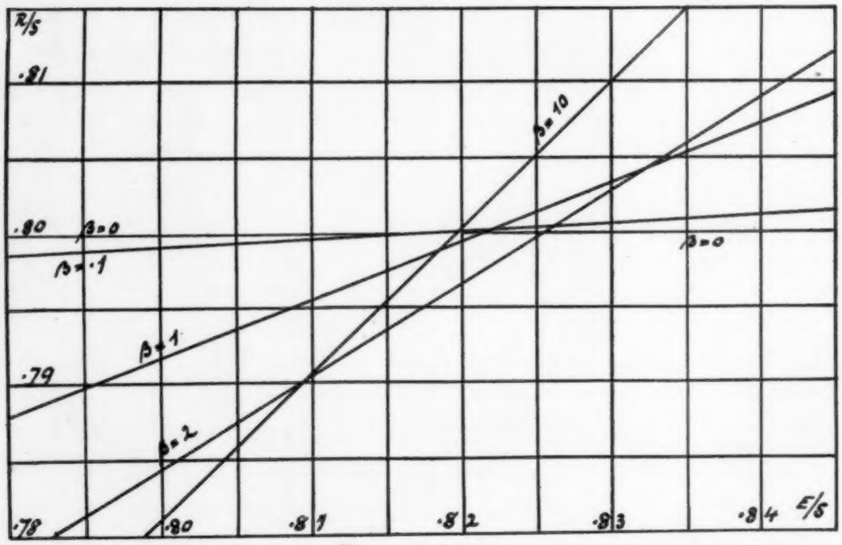


FIG. 3. $st=0.5$.

this case. Thus when E/S is 0.54, a line defined by $\beta=10$, shows an increase in brightness over the background $[(R-R_0)/R_0]$ of 5.4 per cent. and a weaker line ($\beta=1$) gives a deficiency of light of 5.2 per cent.

Table II gives the values of E/S at which the transmitted radiation corresponding to different values of β is equal to that of the transmitted unabsorbed radiation ($\beta=0$). The numbers given define the temperatures of the absorbing medium at which the transition from the dark to the bright lines takes place.

Table III gives in terms of R_0 the intensities of the radiations when the temperature of the absorbing layer is the same as that of the background, the incident light S being in this case considered

to emanate from a black body. The table shows the importance of the effects of scattering on the production of bright line spectra; for, neglecting this scattering, all the numbers would be equal to unity, and we should only obtain the continuous spectrum of the background, the medium not affecting the radiation at all.

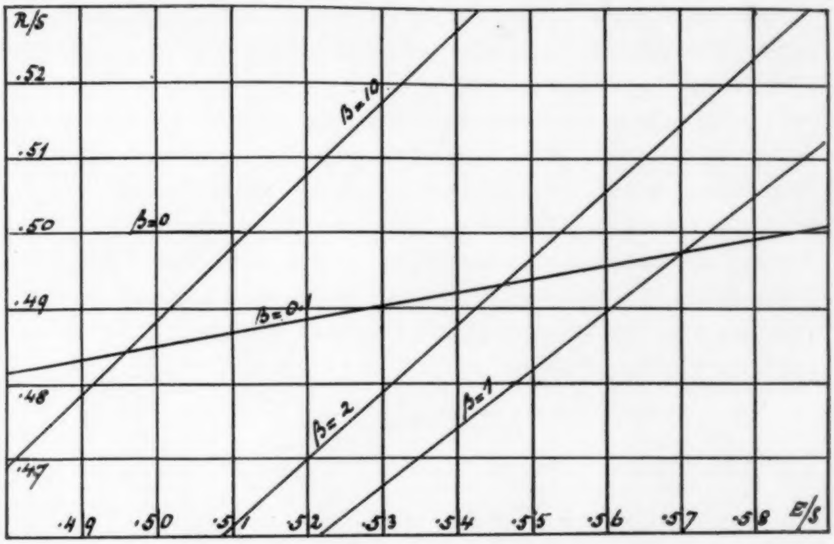


FIG. 4. $st=2$.

TABLE II

	$\beta=$	0.1	0.8	1	1.2	2	10
$st=0.5$...	$\frac{0.8-a}{b}$	0.8149	0.8185	0.8213	0.8231	0.8254	0.8196
$st=1$....	$\frac{0.6667-a}{b}$	0.7065	0.7137	0.7150	0.7155	0.7139	0.6831
$st=2$....	$\frac{0.5-a}{b}$	0.5845	0.5776	0.5736	0.5697	0.5539	0.5123

TABLE III

	$\beta=$	0.1	0.8	1	1.2	2	10
$st=0.5$...	$\frac{a+b}{0.8}$	1.011	1.074	1.086	1.098	1.134	1.220
$st=1$....	$\frac{3}{2}(a+b)$	1.042	1.227	1.257	1.283	1.350	1.464
$st=2$....	$2(a+b)$	1.146	1.604	1.658	1.698	1.794	1.952

8. In the problem as it is usually considered, absorption is introduced by having a cooler medium of uniform temperature in front of a hotter one, and the change of temperature is taken to be an abrupt one. But in considering the phenomena of radiation presented to us by celestial bodies, we must bear in mind that no such discontinuous variation in temperature is admissible. It seems therefore desirable to discuss, even if only in a very simple case, the radiation emitted by a gas of continuously varying temperature.

Equation (6) holds when the temperature variation of the medium is such that the radiation of a black body for the particular wavelength considered varies uniformly with x. We write now for the equation defining the temperature of the medium: $E=j-ux$, where j represents the radiation of a black body which is at the temperature of the external surface of the medium, and u being positive, the temperature increases toward the inside. By differentiation of (6) and the application of (3) we then obtain

$$(B-A)=-\frac{2u}{\kappa+s}+Kae^{(\kappa+s)ax}-K_1ae^{-(\kappa+s)ax}$$

Combining this with (6) we find

$$2A=2(j-ux)+\frac{2u}{\kappa+s}+K(1-a)e^{(\kappa+s)ax}+K_1(1+a)e^{-(\kappa+s)ax} \qquad (17)$$

At a certain distance, for which we may put $x=-t$, we may imagine a radiating black surface to be placed, having a temperature equal to that of the medium at that surface. The incident radiation A must therefore here coincide with the radiation E.

This gives

$$A=j+ut\ .$$

If t is very large, we find by applying (17) to this distant layer

$$0=\frac{2u}{\kappa+s}+K_1ae^{(\kappa+s)at}\ ,$$

which gives for K_1 negligible values when t is sufficiently large.

Putting $x=0$ in (17), we now obtain

$$2R=2j+\frac{2u}{\kappa+s}+K(1-a)\ .$$

If we apply (6) to the case of $x=0$ when $B=0$, $A=R$, $E=j$, we also find, neglecting K_1,

$$R=2j+K\ .$$

Hence, eliminating K,

$$(1+a)R=2aj+\frac{2u}{(\kappa+s)}\ . \tag{18}$$

The character of the radiation is seen from this equation to depend on the relative values of the radiation-gradient and the coefficient of scattering. In order to exemplify the conditions which regulate the nature of the spectrum, draw a plane through the medium at a distance t behind the front surface. Choose t to be such that, owing to scattering and independently of absorption, only 0.8 of the light incident on the layer passes through it. This gives $st=1$. Let $(1+m)j$ be the radiation passing through the plane at a distance t from the front surface. The temperature of the scattering medium varies therefore in such a manner that the radiation of a black body increases in the ratio $(1+m):1$, when $t=1/s$. As the radiation generally is $j-ux$, we have, when $st=1$,

$$j+ut=(1+m)j\ ,$$

or

$$ut=mj\ .$$

If in the second term of (18) we multiply numerator and denominator by t and substitute $st=1$, $\kappa t=\kappa/s=\beta$, we find

$$R=2j\left(\frac{a}{1+a}+\frac{m}{(1+\beta)(1+a)}\right),$$

$$=2j\left[\frac{a}{1+a}+m(1-a)\right] \tag{19}$$

It remains to discuss this equation. For $\beta=0$,

$$R_0=2mj$$

and this may be taken to be the intensity of the continuous background of the spectrum. If the radiative power of any homogeneous radiation is very large, so that β is very large,

$$R_c=j\ .$$

The lines with large radiative power appear therefore bright or dark according as m is less or greater than one-half.

Differentiating (19) with respect to a,

$$\frac{dR}{da}=2j\left[\frac{1}{(1+a)^2}-m\right],$$

we see that as a increases from zero, the radiation increases or dimin-

ishes according as m is smaller or greater than one. A turning-point is reached when

$$m=\frac{1}{(1+a)^2} \ ,$$

or

$$a=\frac{1}{\sqrt{m}}-1 \qquad (20)$$

As a is necessarily positive and smaller than one, this turning-point has a meaning applicable to our problem only when m is greater than one-quarter and smaller than unity. A maximum of radiation is reached in this case, and for values of the coefficient of absorption greater than those defined by (20) the radiation diminishes again. The radiation reaches the same value it has for $a=0$, when

$$R=R_0 \ ,$$

or

$$m=\frac{a}{1+a}+m(1-a) \ ,$$

or

$$m=\frac{1}{1+a} \ .$$

As a is a positive fraction, it follows that in order that $R=R_0$ for a second value of a, it is necessary that m should be greater than one-half.

When there is a maximum, its value is easily found to be

$$R=2j[m+(1-\sqrt{m})^2] \ .$$

We may summarize our results thus:

Case I: $m<\frac{1}{4}$.—With increasing coefficient of absorption, the radiation increases. All homogeneous vibrations appear as bright lines. The brightness of the background ($\kappa=0$) is given by

$$R_0=2mj \ ;$$

that of the brightest line

$$R_c=j \ .$$

Case II: $\frac{1}{4}<m<\frac{1}{2}$.—With increasing coefficient of absorption the radiation increases and reaches a maximum when

$$a=\frac{1}{\sqrt{m}}-1 \ .$$

For greater values of a, the radiation diminishes. All lines are bright, but the lines with the greatest coefficient of absorption are not the brightest. Thus when $m=\frac{1}{2}$, the maximum radiation takes place when $a=0.414$ ($\beta=0.207$), and gives an intensity of $1.172\,f$, while for infinite values of κ the intensity is f.

Case III: $\frac{1}{2}<m<1$.—The intensity rises to a maximum as in Case II, then diminishes until, when $a=\frac{1-m}{m}$, the radiation has the

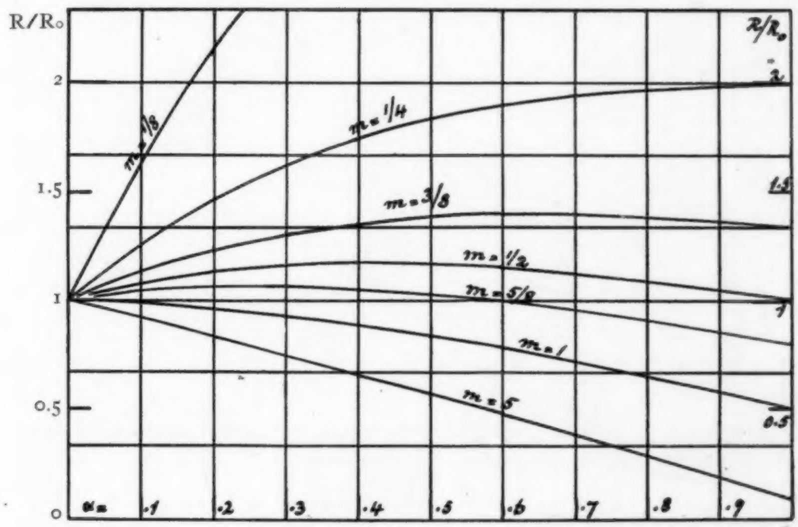


FIG. 5.

same value as when $a=0$. After this point it diminishes. Homogeneous radiations will in this case appear as bright lines when $a>\frac{1-m}{m}$ and as dark lines for smaller greater values of a.

Case IV: $m>1$.—The intensity diminishes continuously with increasing coefficient of absorption. All homogeneous lines appear as dark lines. In Fig. 5 I have drawn the curves which give the intensity of radiation in terms of R_0. The abscissæ represent a and the ordinates:

$$\frac{a}{m(1+a)}+(1-a)\ .$$

Table IV gives the corresponding values of a and β.

TABLE IV

α	β	α	β
0.1	0.0101	0.6	0.5625
0.2	0.0417	0.7	0.9608
0.3	0.0989	0.8	1.7778
0.4	0.1905	0.9	4.2632
0.5	0.3333	1.0	∞

In applying the results obtained, it should be remembered that m defines the "radiation-gradient," which depends not only on the "temperature-gradient," but also on the wave-length. The same increase in temperature will cause a greater radiation-gradient at the violet end than at the red end of the spectrum, and at comparatively low temperatures the radiation-gradient may in the violet be enormously larger than the temperature-gradient. Hence we conclude that with moderate increases of temperature toward the inside of a gas, the lines of a spectrum which have a shorter wave-length are much less likely to be bright than those of longer wave-length.

9. It may help the reader to draw the proper inferences from the preceding results if an elementary demonstration is given showing how bright line spectra are formed in an infinitely thick layer of incandescent gas, which, when the temperature is uniform, should, according to the ordinary theory, give a spectrum identical with that of a black body at the same temperature. In Fig. 6 let an observer look from E at a mass of gas giving out two homogeneous radiations λ_1 and λ_2, differing very largely in their emissive and absorptive powers κ_1, and κ_2 The wave-length λ_1 being that for which the emissive power is great, will chiefly be due to the radiation of a thin layer $L_1K_1K_2L_2$, because waves of the same wave-length coming from behind will be strongly absorbed by it. On the other hand, the vibrations of small emissive, and therefore small absorptive power will be due to the radiation of a much thicker layer. Neglecting in the first instance the loss of light due to scattering, we may draw H_1H_2 so that the layer $L_1H_1H_2L_2$ sends out a total intensity of waves, λ_2 representing the same fraction of the radiation of a black body of the same temperatures as does the layer $L_1K_1K_2L_2$ for

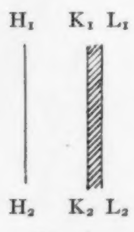


FIG. 6.

the wave-length λ_1. It is well known that whatever be the emissive powers, an indefinite increase in thickness will ultimately give the radiation of a black body. Now, introduce scattering in addition to the absorption. The wave-length λ_1 will not be much affected by scattering, as the light which leaves the gas has only traversed a small thickness of it. On the other hand, the wave-length for which the emissive and absorptive powers are small, being due to the radiation of a thick layer, will be much more weakened by the light scattered, and returned backward. Hence while, in spite of the scattering, λ_1 still shows an intensity nearly equal to that of the black body, the intensity of λ_2 is less. Consequently the radiation will be the stronger, the stronger the emissive power, and hence the gas gives a bright-line spectrum.

In this reasoning it has been supposed that the scattering is of the nature of that due to small bodies and is not a phenomenon primarily dependent on absorption. In the latter case it might be argued that the scattering might be stronger for the wave-length λ_1 in the same proportion as the emissive power is stronger. It is quite possible that a portion of the molecular scattering is selective in character, and, so far as this portion is concerned, our investigation does not apply, without more detailed consideration.

10. I may, in conclusion, briefly indicate the bearing of the results obtained on some problems of astrophysics. It has been shown that a spectrum of bright lines may be given by a mass of luminous gas, even if that gas is of great thickness. There is therefore no difficulty in explaining the existence of stars giving bright lines. The essential criterion which separates the bright-line emission from the dark-line absorption lies in the temperature-gradient of the luminous gas. If the increase of temperature toward the inside of a star is small, bright lines will appear, while the absorption spectra observed in the majority of cases accompany a more rapid variation of temperature. The temperature-gradient is chiefly regulated by the gravitational force, and a star in the early stages of condensation will therefore be in the condition most favorable for the bright-line emission. If the light is but feebly absorbed, so that we can look into considerable depths of the star, it may be possible that the outer regions contribute bright lines, while the hotter inner portions show absorption lines.

The possibility of the simultaneous presence of bright and dark lines of the same element, e. g., of hydrogen, is also strongly indicated by our theory. The matter has already been discussed in sections 7 and 8. The conditions under which the equations of sec. 8 have been deduced are more likely to apply to stars than the conditions of the problem as discussed in sec. 7, and as pointed out at the end of sec. 8, the lines of smaller frequency are those most liable to appear as bright lines. This agrees with the observed facts. The simultaneous appearance of bright lines of smaller and dark lines of greater frequency, has however, also been observed in cases where it is difficult to imagine that scattering plays an important part. I refer to Professor Hale's observations[1] on the spark-spectra observed in liquids. The proper explanation of these and similar observations suggests itself at once, if it is considered that the essential part of the effect of scattering lies in the diminution of the intensity of the continuous background. This diminution is not called for when the body giving the continuous spectrum has not infinitely great thickness and radiates with an intensity less than that belonging to a black body. Putting $s=0$ in (12), we obtain the ordinary equation for the radiation of a body sending out light of intensity S, which before reaching the observers traverses a body which is at a temperature for which the completely black radiation is E, viz.:

$$R=E+(S-E)e^{-\kappa t} .$$

For $\kappa=0$, we have

$$R_0=S .$$

Hence the question of brightness or darkness for a particular wave-length depends on the sign of the quantity

$$R-R_0=(E-S)(1-e^{-\kappa t}) ,$$

and this depends entirely on the question as to whether $E-S$ is positive or negative. If S is the radiation due to a black body at a higher temperature than that of the absorbing body, $E-S$ is necessarily negative and an absorption line will appear. If the radiation S is not that of a black body, but, e. g., a radiation reduced by the same quantity in the red and blue, or even reduced in the same ratio, the peculiar dependence of the radiation curve on temperature and wave-

[1] *Astrophysical Journal*, **15**, 227, 1902.

length shows that $E-S$ which is now positive when the temperature of the two bodies is equal, keeps positive longer with a diminishing temperature of the absorbing layer when the wave-lengths are long than when they are short. I need not enter more fully into this question, because Professor Kayser[1] has fully discussed it in giving what is practically an identical explanation. On applying Professor Kayser's explanation to the case of stars, we meet, however, with the very serious difficulty that we are obliged to consider the radiation of the continuous spectrum which serves as background to be less than that of a black body, which, on the views hitherto held, could not be the case when the radiating body has a great thickness. The consideration of the effect of scattering as explained by the present investigation removes the difficulty. I differ from Kayser in so far that he considers the existence of bright lines in stars to be an indication of high temperature. The small temperature-gradient seems, on the contrary, to argue more in favor of relatively low temperatures. Discussion on these and other connected matters is difficult, however, owing to our ignorance of the relative values of the coefficient of emission κ for different elements, and for different lines of the same element. We do not even know whether in a series such as that formed by the hydrogen lines κ increases or diminishes toward the root of the series.

The appearance of bright hydrogen lines covered by the dark calcium absorption, as presented by the spectrum of *Mira Ceti*, presents no difficulty according to the views of the present investigation. It only implies that the interior of the star has a temperature-gradient insufficient to reverse the hydrogen lines, and an outer atmosphere containing cooler calcium vapor. I consider it indeed as quite possible that, if we could remove the outer layers of the solar atmosphere, we should obtain a spectrum of bright lines.

This brings me to the second consideration suggested by the previous investigation. The prominent part played by the H and K lines of the solar spectrum in stellar atmospheres may be, to a great extent, due to the high values of the coefficient κ. The experiments of Sir William and Lady Huggins show conclusively that when calcium gas is rendered luminous by the electric discharge under

[1] *Astrophysical Journal*, **14**, 313, 1901.

conditions under which the H and K lines can appear, they are most persistent and are seen even when only very minute quantities of the substance are present. We are justified in concluding from these experiments that the emissive power of H and K is very great. The same may be true of other lines characteristic of spark-spectra, and the appearance of these lines in the stellar spectra must therefore be treated with some caution. If a star in its process of condensation increases the temperature-gradient of its outer layers, those lines will first make their appearance as dark lines which have high values of κ. But I must defer the fuller discussion of this matter to another occasion.

The effect of scattering on the intensity of the continuous spectrum of what we call the photosphere of a star may be considerable. When the radiating layer of a gas is sufficiently cool to admit of the presence of particles of solid or liquid matter, of dimensions large compared with molecular dimensions, the reduction in luminosity would take place fairly equally throughout the range of the visible spectrum. There would consequently be no great alterations in the relative intensities of red or blue, and we could obtain a correct idea of the temperature of the radiating body by a thermal comparison of the intensity of radiation in different parts of the spectrum. But when the scattering is molecular, it is sixteen times as large for the extreme visible violet as for the extreme visible red. Consequently the radiation emitted by a mass of gas under these conditions would show the violet considerably weakened as compared with the red. This opens out the possibility that with increasing temperature the violet portion of the continuous spectrum of a star may diminish in intensity as compared with the red end. As will presently appear, we possess some independent evidence that the photosphere emits less violet light than it should do if it were a black surface, but a closer experimental investigation is necessary before this can be definitely established.

I consider that for this purpose the careful investigation of the distribution of intensity in the solar spectrum is a matter of urgent importance. It would be necessary, however, for a satisfactory solution of the problem to measure the intensities everywhere in the intervals between Fraunhofer lines, or, at any rate, to select portions of the spectrum where no prominent Fraunhofer lines are situated.

Unless this is done, we risk that the violet portion of the spectrum should show too small an intensity, merely because it contains a greater number of Fraunhofer lines.

The loss of light by scattering in the solar atmosphere renders it possible for bright lines to appear which are due to vibrations in the front layers, though the temperature at these layers may be less than that which supplies the continuous spectrum. If the scattering is insufficient, its effect may be to obliterate the Fraunhofer line without entirely converting it into a bright line. It is necessary, however, that some kind of law should exist as to which of the Fraunhofer lines are obliterated. The two striking facts to be explained are the absence of the ultra-violet portion of the hydrogen series and the absence of the helium lines. In both cases the lines appear with considerable brilliancy in the so-called chromosphere, and in the flash-spectrum observed at the beginning and end of total eclipses. With regard to the hydrogen series, observations on stellar spectra and laboratory experiments which have already been quoted, would have led us to expect that the ultra-violet lines would be more easily reversible than the less refrangible lines. If the Sun forms an exception, it seems to indicate that the violet part of the continuous spectrum is reduced in intensity relatively to the red portion, and that this reduction is not a mere temperature effect.

This consideration strengthens to some extent the idea that the comparative weakness of the ultra-violet radiation in solar stars is not due to a diminution of temperature. As already mentioned, molecular scattering in the photospheric region might account for the comparative poverty of the more rapid vibrations.

The behavior of helium cannot be due to the same cause, as none of its lines have been seen reversed in the solar spectrum. The correct explanation is, I believe, to be found in this case in the great height to which helium is found to rise above the photospheric layer. The previous investigation has shown that a bright line is more likely to appear when the product of the coefficient of scattering and the thickness of the absorbing layer is large. This may be caused either by a great coefficient of scattering or by a great thickness of the absorbing layer. It is true that this reasoning should apply equally to hydrogen and the metals which rise as high as helium, and I believe

that it does apply. The absence of some of the hydrogen lines in the solar spectrum has already been noted. That the red and blue lines can be seen is no doubt a consequence of the fact that hydrogen exists in much greater quantities than helium, for it should be noted that the helium lines are not bright, but only insufficiently dark to be observed.

This comparative weakness of some Fraunhofer lines which are very prominent in the flash-spectrum, and are probably due to the high temperature of the portion of the solar disk emitting the correspondent radiation, has been commented upon by Mr. Evershed, whose explanation I consider in the main to be correct. Although a further discussion of some points of detail may be desirable, the matter is independent of scattering, and lies therefore outside the range of this communication.

VICTORIA UNIVERSITY,
Manchester, Eng.

THE REVISION OF ROWLAND'S SYSTEM OF STANDARD WAVE-LENGTHS[1]

By LEWIS E. JEWELL.

In considering the desirability of revising Rowland's system of wave-lengths, and of changing to Michelson's absolute values, it is well to consider carefully what were the probable sources of error entering into the determination of Rowland's or Bell's absolute values, Rowland's "New Table of Standard Wave-Lengths," and Rowland's "Preliminary Table of Solar Spectrum Wave-Lengths." It is also desirable that we should consider what work bearing upon this subject has been done since these tables were constructed, and what material has been accumulated capable of furnishing a basis for more accurate tables; and what work it is desirable to do in the near future, in order that the whole subject may be placed upon a satisfactory basis, with the least friction and the least expenditure of energy. In the discussions which have been in progress for some time regarding the corrections to be applied to relative wave-lengths, and the relations between *solar* and *metallic* lines, there are some very important factors which have been almost completely ignored. In the first place, it is well to consider the various errors to which the wave-lengths of lines in the various tables published by Rowland were subject.

The absolute values of wave-length were derived from determinations by Bell, influenced in a measure by determinations of other observers. These were determinations of the wave-length of lines in the solar spectrum, and were corrected, approximately at least, for temperature and air-pressure. Since then better values for the refraction of the air, at different pressures and temperatures, have been determined. The measurements by Bell were also corrected for motion in the line of sight, of the observer's position, as caused by the Earth's rotation upon its axis and revolution around the Sun in a slightly eccentric ellipse.

[1] Paper read in abstract at the Conference on Solar Research, St. Louis, September 22, 1904.

The first determinations of wave-length used in Rowland's early tables were made by Rowland and Koyl, mostly from eye-observations, and were at best only approximate values, and many of the lines taken as standards were of an unsatisfactory character. These measurements were used in the preparation of Rowland's later tables, but were not given much weight, and some of the lines were discarded. Later eye-measurements of relative wave-length were made with both the concave and plane gratings by Rowland and Crew, and afterward by myself from photographic plates taken by Rowland with two or three concave gratings of different values for the grating space. These photographs were upon very fine-grained plates, and the definition in general was very good, but, except in a few cases, no data were given from which the corrections to be applied for temperature and pressure of the air could be derived. Also in the eye-measurements no corrections of this character were made. Rowland himself did not pretend that the measurements were accurate to less than one-hundreth of an Ångström unit, and, such being the case, he had little patience with the idea of making these small corrections; although it is likely that they were responsible for many errors, both systematic and accidental, in his tables, which at times might have been rather greater than one-hundredth of an Ångström unit.

Having made practically all of the measurements from the photographic plates, and the calculations in the work of reduction, I can speak more positively regarding them. They consisted, for the greater part, of two groups of plates, nearly all taken by Rowland, upon fine-grained emulsions prepared by himself. The first set comprised eighteen plates, 14 in. (35 cm) long by 1⅞ in. (4.8 cm) wide, of overlapping spectra for the determination of relative wave-lengths by the method of coincidences, and consisted of a middle strip containing a portion of the solar spectrum in some particular order, and on each side of it strips of a portion of some overlapping spectrum of another order. These were taken by the aid of a shutter which in one position allowed the middle spectrum to reach the photographic plate, and when the shutter was rotated allowed the spectrum of the outside strips to reach the plate, the middle strip being then covered up by the shutter. Rowland was in the habit of giving one of these strips

two exposures, one both before and after exposing the other strip. He did this under the impression that, if anything happened to the apparatus between the first and second, or between the second and third exposures, the average of the first and third would equal the second exposure. This, however, by no means follows. If the effect that produces the error be of a continuous nature, such as the heating of the grating or slit during the exposure or some such cause, that might be true; but if it were due to the act of turning the shutter in some direction and then back again, or some mechanical movement or jar, this would by no means follow, and the third exposure added to the first might simply introduce an error where there had been none before. Also the first change in the shutter might introduce an error in one direction, and the return of the shutter to its original position might introduce an error in the opposite direction, bringing things back to the original condition; and the first and third exposures might thus very well be coincident, but the second exposure would not be compensated for by the average of the other two.

In photographs taken by myself I have found it better to see to it that the shutter worked easily and then to make only two exposures, turning the shutter and doing everything else to avoid any pressure against the apparatus in any direction, and to avoid jars of any kind to parts of the spectrograph. There is no question whatever but that for such work the shutter should be disconnected from the spectrograph, and made entirely independent in its working, or that the parts should be carefully balanced, work easily, and be operated pneumatically or electrically, so that there can be no displacement due to motion of the shutter or pressure against parts of the apparatus. There is no doubt in my mind that in the plates taken for the measuring of coincidences errors of this kind did probably occur, and at times may well have amounted to considerably more than one-hundredth of an Ångström unit. It is quite possible that the errors arising might be such as to cause the lines of the second exposure to be always shifted in a given direction with respect to the other two; and the errors might possibly differ as the camera of the spectrograph was moved to different portions of the spectrum, for even a shift in the position of the observer will cause some variations.

In photographs containing solar and metallic arc-spectra upon

the same plate, the solar spectrum was usually taken in the middle, and the arc spectrum outside. Some of these plates were taken by Rowland, and some of them by myself under Rowland's direction; but scarcely any of them contained data for the determination of motion in the line of sight, or for correcting for atmospheric pressure and temperature, where lines of different orders of spectra overlapped. Another remarkably fine set of plates of the solar spectrum alone were taken by Rowland for making his "Photographic Map of the Solar Spectrum;" but only one of them was used in these measurements, and scarcely any of them had data marked upon them.

The measurements upon both sets of plates mentioned were made by myself, nearly all of them upon a dividing engine constructed by Mr. Schneider under Rowland's direction. The screw of this dividing engine was carefully made by Schneider, and is remarkably free from errors, but the attached microscope was unsatisfactory. The definition of the microscope was not perfect and the field not flat; also some other parts of the dividing engine were not entirely satisfactory. The definition of the plates for coincidences of overlapping solar spectra for the determination of relative wave-lengths were, in general, fairly good, and some of them excellent. The plates for the coincidence of solar and metallic lines were, some of them, good, some bad, and others indifferent. Practically all of them were devoid of data for corrections. One source of trouble was experienced with photographs taken upon thick plates of uneven thickness from the varying focus of different parts of the plate and consequent parallax. Adjustment was made to avoid the trouble as far as possible, and later on means were adopted for nearly getting rid of the trouble. There was, however, always some parallax from curvature of the field of the microscope, but the trouble was avoided and allowed for, as far as possible. The plates for comparison of solar and metallic spectra were 19 in. (48 cm) long by $1\frac{1}{4}$ in. (3.2 cm) wide, and bent to the focal curve of the grating.

Notwithstanding the difficulties mentioned, measurements made upon good lines could, in general, be relied upon to two or three-thousandths of an Ångström unit, and some good lines to one-thousandth of a unit.

All measurements of relative wave-length where lines in different orders of spectra, or solar and arc-spectra, were compared, were subject to the uncertainties due to motion in the line of sight and changes of wave-length due to air-pressures and temperatures, which were not allowed for. Also a few plates were of a very unsatisfactory character, but were used because no others were available at the time.

All of these causes produced errors in Rowland's published wave-lengths which were not allowed for in any way. Also I had discovered a systematic difference in the wave-length of lines in the arc and solar spectra, but Rowland could not be convinced that the difference was due to any cause other than a disturbance of the apparatus between the exposures for arc and solar spectra; and, as a consequence, he had the displacements for different lines upon a given plate averaged up, and applied this as a correction to the arc wave-lengths, to bring them into agreement with the corresponding solar wave-lengths.

This matter has since been thoroughly investigated by myself, and the cause of this difference in wave-lengths determined to be mostly motion in the line of sight of the matter in the Sun's atmosphere producing the absorption lines of the solar spectrum, and to a small extent to a difference of pressure in the atmosphere of the Sun and the electric arc.

For Rowland's "New Table of Standard Wave-Lengths," published in 1893, these measurements were used, together with all previous observations available, and weighted according to their probable accuracy. Then the results were arranged and worked over by Rowland, who introduced certain empirical corrections, where he deemed them necessary, and thus was constructed the table known as Rowland's "New Table of Standard Wave-Lengths." In 1896 Rowland published in the *Memoirs of the American Academy of Arts and Sciences* an extended account of the methods used by him in the construction of this table.

Later I measured up the complete solar spectrum upon Rowland's plates which were used for the production of "Rowland's Photographic Map of the Solar Spectrum." These plates have already been referred to as of particularly good quality.

Considerable difficulty was experienced during these measure-

ments because of Rowland's ideas concerning the necessary accuracy of such measurements, and the desirability of measuring all of the lines in the spectrum. Rowland at that time was not much interested in line-of-sight work, and, in fact, work of that kind was not far developed at the time, and he saw no need of an accuracy greater than was sufficient to identify solar with metallic lines, particularly as he did not consider the wave-lengths of the lines in his "New Table of Standard Wave-Lengths" to be more accurate than one-hundredth of an Ångström unit. Also he did not favor the measurement of the very faint lines of the solar spectrum which were difficult to see, and, in fact, he disbelieved in the existence of many of them. He also disapproved of the measurement of more than one, or at the most two, standard lines at each end of a set of measurements, in order that measurements made at different times might be properly connected, saying that an accuracy greater than one-hundredth of an Ångström unit was entirely useless where the measurement of all of the solar lines were concerned, as the standard lines did not have an accuracy greater than that. Upon these points we were not in agreement at all, and as a result a sort of compromise was arrived at.

Before the measurements were undertaken, Rowland had me take a series of photographs of the solar spectrum upon commercial photographic plates which were rather coarse-grained, and as a consequence many faint and fine lines were obliterated. This he seemed to think a desirable feature, but another important consideration was that these plates were upon the same scale as the dividing-engine screw, and all of a uniform scale, whereas neither was true of the map plates. It was only when he was convinced that the uncertainties of interpretation in the case of double and multiple lines, and the difficulty of measurements in general, would require a longer time to measure the lines upon these plates than the clean, sharp lines upon the fine-grained map plates, that Rowland consented to discard them, and have the measurements made upon his remarkably fine map plates. I had also discovered a method of reducing the measurements which required very little more work than the plates made to scale.

Finally these plates were measured with some few of the most difficult faint lines omitted, but with their existence and their approximate positions indicated. Some of these lines have since been

measured, and the measurement of the others will present little difficulty. The lack of measurements upon a greater number of good standard lines was perhaps the greatest defect of this complete table of solar lines. Of course, the standards were measured along with the other lines, but upon plates containing hundreds of lines, and in some cases two or three thousand lines, they could not all be measured at one sitting; and it was the measurement of more than one or two standards where the different sets of measurements overlapped, to which Rowland objected. These defects can be remedied, however, without much difficulty by remeasuring at one setting, not only the lines taken as standards, but other good clean lines upon these plates. It is the intention to do this as soon as possible, and also to measure the omitted faint lines.

Notwithstanding the difficulties encountered in the making of this table of the complete solar spectrum, known as "Rowland's Preliminary Table of Solar Spectrum Wave-lengths," the wave-lengths not only of the standard lines, but also of most of the good lines in the spectrum, are probably more accurately given than the wave-lengths of the lines in the "New Table of Standard Wave-Lengths."

Later I became interested in the subject of the spectroscopic determination of the rotation period of the Sun, and especially in the question as to whether the period of rotation varied with the elevation above the Sun's surface. This investigation was undertaken largely as a result of the discovery that a line in the solar spectrum had a somewhat composite, and in a measure a complicated, structure; the different portions of shaded lines being produced by absorption at different heights in the solar atmosphere. This investigation was carried on for more than a year, and a great many photographs containing the comparison spectra of the opposite limbs of the Sun, or of one limb and the center of the Sun's disk, were taken. This work was mainly confined to the ultra-violet and violet portions of the spectrum between λ 3700 and λ 4200; but some few plates were taken from the extreme ultra-violet to the yellow and orange parts of the spectrum.

This work was done with the utmost care, and also many sets of eye-measurements were made. These eye-observations included measurements of certain lines in the yellow portion of the spectrum,

near the D lines, which are due to water-vapor in the Earth's atmosphere, and have proved to be remarkably accurate, the water-vapor lines serving as an excellent check upon any possible motion of the apparatus during the observations. Also comparisons were made of a number of metallic with corresponding solar lines, using both the naked arc, and the arc under various pressures. These observations have proved to be very satisfactory, more so than similar measurements upon photographic plates. Many photographs of arc and solar spectra have also been taken, in which all necessary data for corrections have been included.

In the reduction of observations for the determination of the rotation period of the Sun, it was soon found necessary to determine, as accurately as possible, the relative wave-lengths of an entirely new set of standard lines throughout that portion of the spectrum included in the investigation. Some of the standard lines included in Rowland's table were found to be of an unsatisfactory character, and the relative wave-lengths of all of them were found not to be sufficiently accurate for my purposes. The lines which I selected (some of which were the Rowland standard lines that were of good character, or particularly useful for other reasons) were carefully chosen for their sharpness, in both solar and arc-spectra, or for their theoretical interest, as indicated by the investigations mentioned.

The plates taken for the determination of the Sun's rotation period have been carefully measured, all of the better plates in reversed direction as well as direct, and a few of the most important plates have been measured twice both ways. A considerable portion of the work of reduction has been made, but as there were over a hundred plates measured with over a hundred lines upon most of them, and considerably more than that upon a few of them; and as the lines occurred upon both a central and the two outside strips, requiring two sets of measurements, both direct and reversed, the measurements and the necessary work of reduction required a great deal of time. In addition to this, many plates having both arc and solar spectra together were measured, and also many plates having only the solar spectrum.

Besides the plates mentioned, there are a considerable number yet which it is desirable to measure, and it is important that Rowland's

map plates should be gone over again, and all of the selected lines mentioned, occurring upon them, should be carefully measured in both directions. This has been done for some of the plates, but should be done for all of them; and the plates containing arc and solar spectra in the other regions of the spectrum should be measured so as to have an unbroken series of selected lines throughout the entire range of the spectrum. When this is done, the relative wave-lengths of a large number of carefully selected lines in both solar and arc-spectra will have been determined with an accuracy considerably greater than that found in present tables. Moreover, the motion of ascent or descent in the Sun's atmosphere, and the pressure of the Sun's atmosphere where the absorption producing the solar lines takes place, will have been determined for many of the lines and data obtained for determining it approximately for all of them. Much of the work of reduction has already been done for the plates measured, but, as I have had no assistance of any kind, and for the past few years the pressure of other work has prevented my finishing it, there is a great deal to be accomplished yet, but it will be done as soon as possible.

When this work is completed, there will be available a set of carefully selected solar lines whose relative wave-lengths are as accurately determined as is possible at present, and also the wave-lengths of the corresponding arc and spark-lines. The various relations between them will have been determined as well as conditions admit, and it is also expected that the intensities of the lines will have been determined upon a rational system depending upon accurate measurements or comparisons. The basis of the system of comparisons is the varying intensities of the first line in the tail of the a group due to oxygen in the Earth's atmosphere ($\lambda = 6287.953$); and the scale of intensities used is a quantitative one easily reproduced at any time. The actual comparisons, however, are to be made by the use of a photographic scale constructed by a method which has proved satisfactory in actual use. This will require a little, but not much, experimental work, and the actual comparisons are yet to be made; but measurements already made by this method have shown it to be entirely satisfactory.

All of the work mentioned can be accomplished without much

difficulty, and when done it should be based upon the best determinations of absolute wave-lengths available.

Although without doubt Michelson's determinations of absolute wave-length are more exact than any others, it would be desirable to have them repeated by other observers with somewhat different apparatus, and it might also be well for Michelson to repeat his measurements under somewhat different conditions.

It might also be desirable to make absolute determinations of wave-length with larger and better gratings than have ever been employed heretofore for that purpose. If used with a parabolic reflector and a flat mirror in such a manner that the parabolic reflector both collimates the light and brings it to a focus, the flat grating could be made to possess advantages not had by gratings with collimators and objectives; as the focus for overlapping spectra would be identical, and photographic plates could be used in the work.

The grating should be ruled so as to avoid, if possible, errors affecting measurements by careful selection of the ruling diamond and its adjustment; and, if large gratings are made for this purpose, an effort should be made to eliminate to a greater degree than heretofore the periodic and other irregularities of ruling. I believe that this can be done.

The grating method, if it can be used with better and larger gratings, and the other parts of the apparatus better than used heretofore, has many advantages not possessed by the interferometer, in that sharp lines in both arc and Sun can be used, with which better comparisons can be made than with the cadmium lines used by Michelson, which in the arc are not so satisfactory as many other lines. Also photography can be used, and absolute determinations be made in the ultra-violet as well as in the visual spectrum.

More than one large grating should be used, and measurements should be made on both sides of the normal, with each of them. Moreover, "freak" gratings should not be used, but only gratings so ruled that the spectra are symmetrical. Such gratings can be made, and no others should be used for the purpose.

It seems to me that, although it is desirable to have more accurate determinations of relative wave-lengths than are at present available, and also desirable that they be based upon the most accurate absolute measurements which it is possible to make, it would be a serious mistake to make any change now, without having the matter more

thoroughly threshed over than it has been so far; so that what is finally adopted by scientific men will not in a few years be in need of adjustment. Also it seems to me that the set of selected lines which has been mentioned might be made the basis of a set of standard wave-lengths, with the addition of other lines, principally arc or spark-lines, which in various researches have proved to be useful for reference purposes. The relationships between the solar and corresponding metallic lines should all be determined as accurately as possible. Thus we should have a working set of standard lines whose wave-lengths are given for both solar and arc or spark-spectra, with their various relations to each other, and their relative wave-lengths sufficiently accurate for all astrophysical work likely to be done. Furthermore, a specially selected series of some twenty-five or fifty lines scattered throughout the spectrum, which are of good character in both solar, arc, and spark-spectra, should have their relative wave-lengths determined with the utmost possible accuracy with one or more concave gratings, of longer focus, larger size, and better quality than heretofore used for that purpose. These gratings should be at least of thirty or forty feet radius, and be mounted in a thoroughly satisfactory manner, in a place where the work can be done without interference from street traffic or other such troubles. The wave-lengths of this comparatively small number of lines should be thoroughly tied together by the method of coincidences. Two or three gratings having different values for the grating space should be used, the work done by photography, and the measurements made by a dividing engine with an accurate screw made for the purpose. The concave grating possesses advantages for such work not possessed by any other instrument, but the lines in the visual position of the spectrum should also be determined by the interferometer, both as a check and because it will probably give more accurate results in the visual spectrum than the grating. But dependence should not be placed upon the one form of instrument, until results with both have been obtained under the best possible conditions, and then carefully compared with each other.

Finally, the wave-lengths of the few lines referred to, which can be called standards of reference, should perhaps be made to depend upon absolute measurements by interferometer methods, of the red cadmium line, or a few other lines measured in the same way.

How accurate the absolute measurements can be made, it is difficult to say; but the relative wave-lengths of the standards of reference can probably be measured to considerably less than one-thousandth of an Ångström unit. The relative wave-lengths of lines in the working table can probably be made accurate to nearly one-thousandth of an Ångström unit for the better lines, while the others should not be much more in error.

When these sets of standard lines are placed upon a satisfactory basis, the measurements of the wave-lengths of all of the lines in the solar spectrum can be readily reduced to these new values and the few omitted lines added. However, some new plates should be taken in the ultra-violet spectrum above λ 3100 extending as far as possible, and be carefully measured, since Rowland's and other plates of the solar spectrum in this region are unsatisfactory. These photographs should be made on plates as little sensitive as possible to green and yellow light, and with a grating free from diffused light. The photographs should be taken at a station having a considerable altitude above sea-level.

Plates in the extreme red, as far as possible to photograph, should also be taken, or Higgs' original negatives should be measured in the same manner as Rowland's map plates have been.

Also all estimates of intensity should be reduced to the rational system referred to. The full table giving the wave-lengths, etc., of all the lines in the solar spectrum can then be published with the corrected identifications.

All of this work cannot be done in a day, and it cannot all be done by one person; but it seems to me that it would be a mistake to make any radical changes in the wave-lengths of lines until this work has been done as far as possible; and it ought not to take very long to accomplish this result, if it is gone about in the right sort of way.

In the meantime a table can be constructed and published, giving the necessary factors for reducing the wave-lengths in Rowland's published tables, to the absolute values determined by interferometer methods; but as Rowland's standards are not entirely satisfactory in some other respects, any final change from his present system should be avoided until a better table of standards can be published.

Johns Hopkins University,
Baltimore, Md., September 1904.

THE BRUCE PHOTOGRAPHIC TELESCOPE OF THE YERKES OBSERVATORY

By E. E. BARNARD

My experience at the Lick Observatory with the Willard portrait lens impressed me with the importance of that form of instrument for the picturing of large regions of the heavens.

That lens, which was purchased at second hand from a photographer in San Francisco, was made for, and originally used in, taking portraits—from which fact its name has come. These large short-focus lenses were necessary in the days of wet-plate photography to gather a great quantity of light and to give a brilliant image to lessen as much as possible the time of sitting. But when the rapid dry plates came into use these lenses were no longer needed, and much smaller, more convenient, and less expensive lenses took their place. The great light-gathering power for which they were so valuable in the wet-plate days make them specially suitable for the photography of the fainter celestial bodies. They were made on the Petzval system and consisted of two sets of lenses, from which fact they are also called "doublets." In this paper I shall refer to them mainly as portrait lenses, as that name appeals more directly to me.

The main advantage of the portrait lens lies in its grasp of wide areas of the sky and its rapidity of action—this last result being due to its relatively short focus. The wide field makes it specially suitable for the delineation of the large structural details of the Milky Way; for the discovery and study of the great nebulous regions of the sky; for the investigation of meteors and the determination of their distances; and especially for the faithful portrayal of the rapid changes that take place in the forms and structures of comets' tails. There is other and important work where this instrument has shown its special adaptability; viz., in the discovery of asteroids and comets and variable stars; and when it becomes possible greatly to extend its field of view without lessening its rapidity, it will be of the greatest value in the study of the zodiacal light and the gegenschein—two

mysteries that perhaps for their explanation are only awaiting such a photographic investigation.

The portrait combination is not intended in any way to compete with the astrographic telescopes, or with any of the larger photographic refractors or reflectors. It must be considered as supplemental to these, because their limited field confines them to small areas of the sky. There is a great and valuable work for these larger telescopes, however, in the accurate registration of the places of the stars, for parallax, and, in the reflector, for depicting the features of the well-known nebulæ, etc.

There is, I think, however, a question as to the most advantageous size for a portrait lens, and I have believed that the best results can be obtained with an instrument of moderate size; or, in other words, I believe that a portrait lens can be made too large to give the very best results, just as it can be too small. It is also true that both large and small portrait lenses are individually valuable. There is a kind of supplementary relationship between them. The small one will do work that the large one cannot do; and the reverse of this is equally true; for though the small one is quicker for a surface—such, for instance, as the cloud forms of the Milky Way present to it—the larger one, mainly on account of its greater scale, will show details that are beyond the reach of the smaller one. Another important fact is that as the size of the lens increases the width of the field rapidly diminishes, and width of field is one of the essential features of the value of the portrait lens.

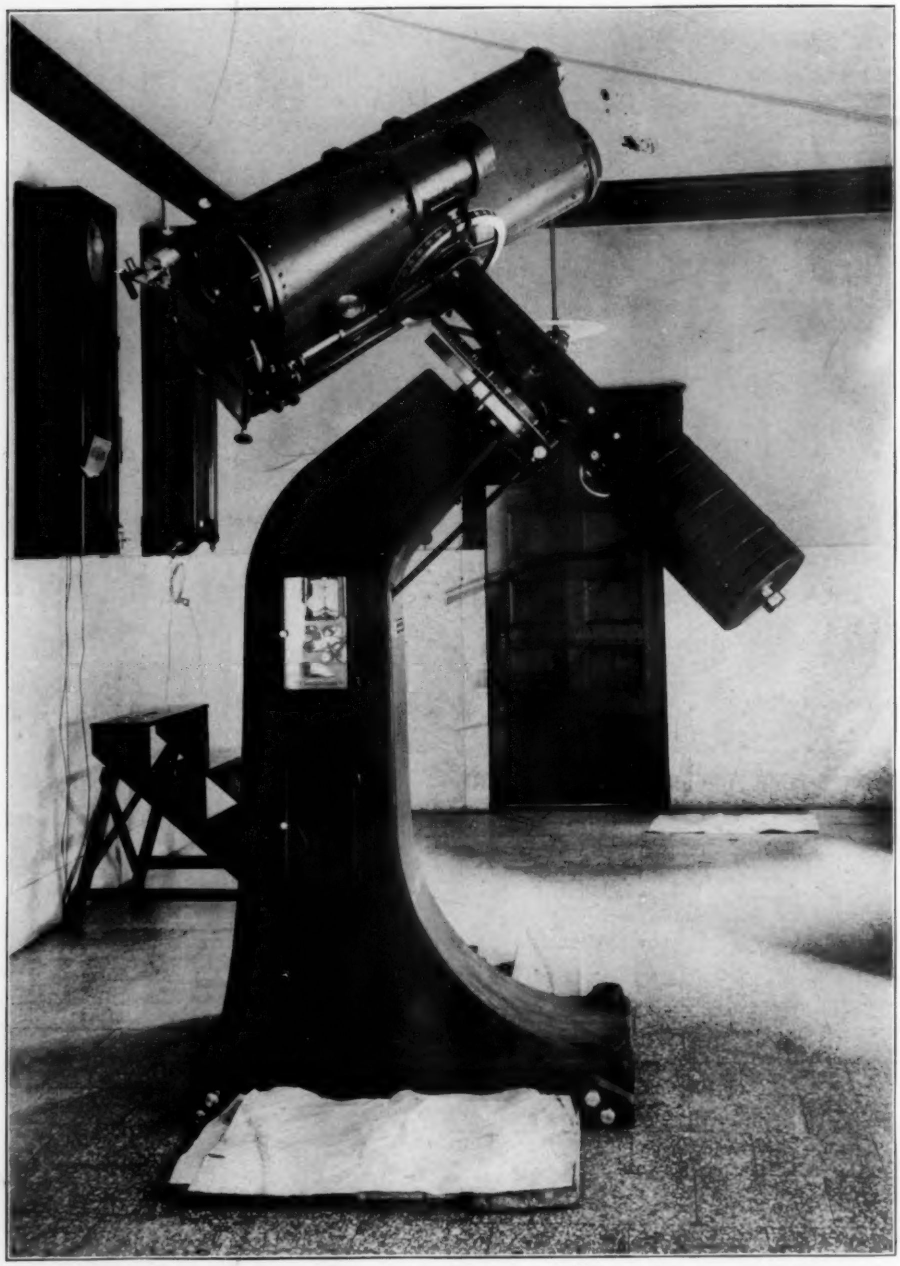
There would, therefore, seem to be a happy mean, when the available funds limit the observer to one lens only.

As a matter of experience, it has seemed to me that a lens of the portrait combination about ten inches in diameter would best serve the purpose of the investigations that have just been outlined.

For several years I had tried to interest someone in the purchase of such a lens, but without success. Finally I brought the matter before Miss Catherine W. Bruce, who had done so much already for the advancement of astronomy. In the summer of 1897 Miss Bruce placed in my hands, as a gift to the University of Chicago, the sum of $7,000 for the purchase of such an instrument and for the erection of a small observatory to contain it.

The instrument and observatory have both been completed and put

PLATE I



Bruce Photographic Telescope in Corridor of Yerkes Observatory

in working order during the present year. The instrument consists of a five-inch guiding telescope and two photographic doublets of 10 and $6\frac{1}{4}$ inches aperture, rigidly bound together on the same mounting. An unusual delay was produced by my anxiety to get the best possible lens for the purpose.

The long exposures demanded in the work of an instrument of this kind require an unusual form of mounting to give an uninterrupted exposure. The mounting of the Willard lens was an ordinary equatorial and was not made specially for it. It did not permit an exposure to be carried through the meridian, except in southern declinations. This was a great drawback since in a long exposure it was necessary to give all the time on one side instead of dividing it up to the best advantage on each side of the meridian.

There were two forms of mounting in use that would permit a continuous exposure. These were (1) the English form of equatorial mounting, which is a long polar axis, supported at each end with the tube swung near the middle; (2) the Potsdam astrographic equatorial, in which the polar axis projects far enough to allow the telescope to swing freely under the pier. Neither of these mountings has appeared to me to be entirely the best form for the purpose.

The focal length of the 10-inch is 50 inches (128 cm); with this short length it seemed that if the pier itself were bent to form the polar axis the telescope could be made to swing freely under the pier in all positions of the instrument. With this idea in view, I went to Cleveland to confer with Messrs. Warner and Swasey on the matter. Mr. Swasey at once took the deepest interest in the proposed telescope, and eventually evolved the scheme that was ultimately adopted in the mounting. The result was entirely satisfactory, and the mounting is, I believe, the best for the purpose that has yet been made.

The next question was the lens, and here is where the delay occurred. It was my wish to get the widest field possible and the shortest relative focus consistent with such a field. This proved to be a problem of the most extreme difficulty. Dr. Brashear, who was appealed to for the optical part, entered heartily into the subject. So earnest was he in his endeavors to fulfil the required conditions that he made at least four trial lenses of four inches' diameter and upward. But my ideal was evidently too high and one not attainable with optical skill.

In the interests of the matter I made a visit to Europe to see if better results could be had there, but, in the end, it proved that Brashear's lenses more nearly fulfilled the requirements than any that I saw elsewhere.

In the meantime Mr. Brashear, with characteristic faith in his skill, ordered the glass and made a 10-inch doublet on his own responsibility. This lens gave exquisite definition over a field some 7° in width and could by averaging be made to cover at least 9° of fairly good definition. Though this did not come up to the width of field originally proposed, it was finally accepted, as it seemed the best that could be obtained.

Dr. Brashear has supplied me with the following information about the ten-inch lens for the present paper.

The glass disks were made by Mantois, of Paris, and delivered to Brashear in May of 1899, and the lenses were completed in September 1900.

"The general construction is that which was first found by Petzval years ago, and has proven itself quite the best where great angular aperture with sharp definition is imperative. The curves have been somewhat modified from our experience in the construction of other lenses—particularly of those made for Dr. Max Wolf, of Heidelberg, Germany. It departs, however, from the ordinary practice of opticians in being corrected for short wave-lengths of light. This would be quite objectless in a camera which is to be used for portraits, but is not without moment in astronomical photography.

"The materials employed were specially chosen for their transparency—the flint being very light and the crown very white. The focal lengths of the front and rear combinations are in a ratio of about 7 to 12, while the focal length of the system is very nearly five times the aperture. The focal length you may find very slightly modified; indeed, it is our custom to balance the inevitable zonal differences of magnification, which difficulty is found the most formidable to all constructors of astronomical photographic objectives."

The accumulation of interest had by this time permitted the purchase of a 6¼-inch Voigtländer lens of 31 inches (79 cm) focus, which had been in commercial use, and a new 4-inch Voigtländer lens with the remarkably short focal length of 7.9 inches, having a focal ratio of less than ½.

As indicated, the telescope is really triple in character, there being

three tubes bound rigidly together on the same mounting—the 5-inch visual telescope for guiding, and the 10-inch and $6\frac{1}{4}$-inch photographic doublets.

The focus of the 10-inch, determined from the photographs, is 50.3 inches (127.8 cm), and the scale is therefore 1 inch $= 1^\circ.14$ or $1^\circ = 0.88$ inch. The ratio, $\frac{a}{f} = \frac{1}{5.03}$, I believe to be the best for the purpose.

In the matter of a guiding telescope the limited means would not permit of anything larger than 5 inches, which is sufficiently powerful for ordinary purposes, though for the photography of comets a larger one would have been desirable. The guiding telescope I used with the Willard lens at Mt. Hamilton was only $1\frac{3}{4}$ inches in diameter. Of course, the question of a double-slide plate-holder was considered; but in a small telescope like this the tubes are so rigidly bound together that such a device is not necessary to insure faithful guiding. Furthermore, for work of this kind the double-slide plate-holder would be seriously objectionable.

The plate-holder for the 10-inch carries a plate 12 inches square, while the one for the $6\frac{1}{4}$-inch carries a plate 8×10 inches.

A high-power eyepiece is used on the 5-inch for guiding in conjunction with a right-angled prism. This is more convenient than direct vision, especially when photographing at high altitudes. The eyepiece has an adjustable motion to the extent of 2° in any direction, thus insuring the finding of a suitable guiding star. This is also valuable in photographing a comet, as it permits the displacement of the comet's head to one side of the center of the plate, thus securing a better representation of the tail.

Two spider-line cross-wires in the eyepiece are used for guiding. They are illuminated by a small electric lamp by the aid of two small reflecting surfaces which throw the light perpendicularly on the wires. The intensity of the illumination is readily regulated. By this means almost the smallest star visible in the 5-inch can be used for guiding purposes.

The illustration will give a better idea of the Bruce telescope than any mere words can do. Indeed, there are very few things about it that need explanation. One feature, however, will not be clear without a description, viz., the method of adjustment for latitude in case the

telescope were removed to a different latitude. It was intended that the instrument should be portable when occasion required, for the purpose of observing eclipses, etc., and for possible transportation to the Southern Hemisphere.

The pier really consists of two parts. Just above the clockroom it separates into two pieces which are bolted together on the inside of the pier, and hence no break appears in the continuity of the pier.

For change of latitude, it is only necessary to insert a wedge-shaped section between these two parts of such an angle that it will produce the required change of latitude. This ordinarily would necessitate only a slight change in the length of the driving-rod which is adjustable. No other means of adjustment seemed feasible.

As it was possible that the instrument might some time go to the Southern Hemisphere, Messrs. Warner and Swasey were asked to insert some sort of gearing that would readily permit of a reversal of the motion of the clock. The device they introduced is extremely simple and efficient. In a couple of minutes' time the motion can be changed from west to east. At the point where the driving-rod joins onto the worm-screw for driving the worm-wheel carrying the telescope, the small gear-wheel which makes the connection can be reversed and placed on the other side of the gear-wheel at the end of the driving-rod; this will reverse the direction of motion of the worm-wheel and hence of the telescope.

The telescope is supplied with fine and coarse right ascension and declination circles, the fine circles are divided on silver and are read by verniers.

The slow motions for guiding are brought down conveniently to the plate-end of the instrument.

For each of the photographic lenses there is an inner tube, with focusing scale, which can be racked back and forth for the adjustment of focus.

For adjusting the instrument in position, the base of the pier rests on two broad iron bars at the north and south with screws at their ends. The upper screws in the picture are for changing the azimuth, while the lower ones push iron wedges that elevate the northern and southern ends of the pier. This latter arrangement seems to be about the only thing for criticism about the instrument. The wedges are

not attached to the screws, and hence if they are pushed in too far they cannot again be drawn back and the other end of the pier must be raised; and if by chance this goes too high, it is necessary to come back to the first wedges and push them higher again. If they were attached to the screws, they could be drawn back again. As the wedges are rather thin, one or two misses of this kind drives them in so that they can no longer act. It would have been better to have had the wedges fastened to the ends of the screws, so that by turning the screws one could either raise or lower the end of the pier. But this is a mere detail that can be easily remedied in future instruments. It is a question if the old method of adjustment in altitude by vertical screws is not a better one than this—certainly it would be preferred to the present arrangement. The pier is very heavy, weighing some 1,200 or 1,300 pounds (550–600 kilos). This great weight is necessary to support the overhanging mass of the telescopes and the top of the pier.

The driving-clock is of Warner and Swasey's regular conical pendulum pattern, which by all means seems to be the best form of driving-clock. It is a beautiful piece of mechanism and performs satisfactorily, though we intend to introduce an electric control for work with it hereafter.

The photograph shows the telescope as it was temporarily set up in the corridor of the Yerkes Observatory.

As will be seen, the design is a new one, and although Messrs. Warner and Swasey have made at least one mounting of this kind (for the Tokyo Observatory) before the Bruce telescope was commenced, it was made from their design for the present instrument, so that the Bruce is the original of this particular form of mounting.

The photograph shows the compact and rigid form in which the tubes are mounted, and it will at once be seen how the combination can swing freely under the overhanging pier.

The instrument was finally finished and placed in position in its observatory in April of 1904. The results so far obtained with it have proved satisfactory and give promise of a useful career.

As I have said, small portrait lenses have their special advantages as well as the larger ones. Where it is possible, it is desirable that two or more lenses should be used on the same mounting, a very

important point being that they mutually verify each other. Duplicate lenses would not seem to be either the most economical or the best arrangement. In that case they would serve only as a verification and could have no other value, unless indeed one of the plates should meet with an accident or be defective—circumstances that would not be of sufficiently frequent occurrence to justify the extra outlay. The best plan would seem to be to have one of the instruments decidedly different from the other so that an independent series of pictures of the same region could be secured on a very different scale. Photographs with these, at the same time that they mutually verified each other, would have other values peculiar to themselves.

The 10-inch and the 6¼-inch, therefore, mounted together, give a very desirable variety in respect to scale, at the same time that the 6-inch is sufficiently powerful to be an almost perfect verification of anything the 10-inch may show. There is plenty of room for other and smaller instruments to be fastened onto the mounting or tubes, and it is intended to utilize this space, especially during periods when meteors are plentiful, or in the case of a bright comet.

The very short-focus Voigtländer lens mentioned is very rapid. It is indeed so rapid that the sky itself photographs with it, which is a serious disadvantage. Under ordinary conditions the sky is more or less whitish, due to the scattering of starlight by dust and moisture in the air; this milkiness strongly affects the plate with a very quick lens such as the one I refer to, and a long exposure is, therefore, impossible; at the same time, for want of contrast the very objects for which such a lens is intended are lost in the general illumination of the plate. To use an instrument of this kind with any success, a pure atmosphere, free from any whiteness, is required. This lens gives a little over one inch, or about 11° or 12°, of perfect definition. Experiments show that plates slightly concave will readily double this amount, and it is hoped some time to use curved plates with it. At present the trouble in using curved plates comes entirely from the difficulty of getting them uniformly coated with the sensitive emulsion. This is a fault that the plate-makers would readily remedy if there were any commercial demand for such a plate that would pay them for their trouble.

In the actual working of the instrument, some few alterations were

necessary, but through no fault of the makers. These changes have been very skilfully made by Mr. Johannessen, the instrument-maker of the Yerkes Observatory, to whom I am very greatly obliged.

The Bruce Observatory is a wooden building of size, 15×33 feet, with the greater length lying east and west. The dome, which is central, is 15 feet in diameter and revolves on 8-inch roller-bearing iron wheels which were supplied by William Gaertner & Co., of Chicago.

In the use of any telescope a wide observing slit is either a necessity or a great convenience. Too often this opening is entirely too small for the telescope used. Besides the inconvenience it produces, such a small opening is often productive of poor seeing.

The large field of the Bruce telescope made a wide opening in the dome a necessity. It was therefore made 4 feet wide, which seems ample for all purposes. The telescope rests on a brick pier, and the observing-room is reached by a small stairway against the inner south wall of the building. Below the observing-room and around the pier is an octagonal hall. This is entered from the outside by a door to the north. The east end of the building consists of one room, which may be called the office or library, leading into the hall just mentioned. The west end of the building consists of two rooms, one of which is entered from the hall, and from this room a door leads into the other, which is a dark room with sink and water pipes.

The octagonal form of the walls supporting the dome conveniently gives space for four closets in the east and west portions of the building. These are very useful, especially the one in the dark room, as it makes a good temporary storage-room for plates that are being used. The two closets in the east room are respectively intended for clothing and for books.

As the observations are often carried through the zenith with this telescope, the floor of the observing-room would need to be very low, or the observer must assume a crouching and uncomfortable position for a portion of the exposure. If the floor were low enough for comfort in this position, the observer would have to be perched upon a high observing-chair for a large part of the time, which would also be inconvenient. To modify these conditions as much as possible,

a couple of small, removable trap-doors are let into the floor where the most awkward position of the telescope occurs beneath where the pier overhangs. The space between the floor and the ceiling of the room below—or the top of the brick pier—is about 15 inches. In the lower position of the instrument one can remove the cover from a trap-door and sit comfortably on the edge of the floor with his feet in the space below and guide with the greatest ease, as a diagonal eyepiece is used. With this arrangement the floor has been placed at a convenient height for the observer in all positions of the instrument, and at most only a small low observing-chair is needed. This is a simple and great convenience in using an instrument of this kind. There is also in the floor on the west side of the pier another and larger trap-door through which the pier can be lowered when it is to be transported elsewhere.

No instrument was available for laying out a meridian when the Bruce Observatory was built. It was necessary, therefore, to find the meridian with the naked eye. As the way this was done may be helpful to others in a similar predicament, I will briefly describe it.

The Central Standard time of the passage of the lower meridian by η *Ursae Majoris* was computed and the watch set to correct time. At a point 400 feet south of the 40-inch dome a plumb-bob was suspended with a fine white cord from the hand. This cord was illuminated by the light from a bull's-eye lantern. At the exact moment the star crossed the meridian, as indicated by the watch, the star and plumb-line were made to coincide with the vertical wall of the tower of the 40-inch dome, and the plumb-bob was dropped, and a thin stake driven vertically into the ground where its point had touched. This stake and the edge of the tower were then in the meridian. On another night, as a check, the process was repeated at a distance of 200 feet. These two vertical stakes were found to be so exactly in a line with the edge of the tower that the eye could detect no difference.

This meridian line was then carried westward by careful measurement to the proposed site of the Bruce Observatory. As a further check, after the pier had been built, a string was suspended, with a weight on the end, from a support above the pier and at the computed time of the transit of the Sun the shadow of the string was traced on

PLATE II

The Bruce Photographic Observatory of the Yerkes Observatory
April 15, 1904. View from Northwest

the top of the brick pier. So exactly had the pier been built that scarcely any deviation existed between its side and the trace of the shadow. Furthermore, when the telescope was put in position and adjusted, its position scarcely sensibly deviated from the adopted meridian.

A better way to have laid out the meridian at the site of the observatory itself would have been to suspend a plumb-line from a support several hundred feet to the north, with a light thrown on it to make it visible, and to have used it instead of the edge of the tower of the 40-inch dome. This star η *Ursae Majoris* transits the lower meridian at a very low altitude here, which makes it very convenient for the purpose of getting the meridian by the above method.

This small observatory is a beautiful building, as will be seen by the photograph. It is the design of Mr. James Gamble Rogers, architect, Chicago. The style is plain and simple. The body of the building is a light gray, with white trimmings, and the dome is white. The center of the dome is 139 feet west and 394 feet south of the center of the 40-inch dome. It lies between the Yerkes Observatory and Lake Geneva, which it overlooks and which is seen in the background of the picture. The horizon is free in all directions, except an unimportant part cut out by the 40-inch dome. The building is lighted by electricity, and it was intended also to heat it by electricity, but the expense of such heating perhaps makes it prohibitive. The electrical appliances were all put in by Mr. Frank Sullivan, of the Yerkes Observatory.

From the numerous photographs already made with the Bruce telescope I have chosen two pictures of the Milky Way, one of which was made with the 10-inch, and the other with the $6\frac{1}{4}$-inch. These pictures are both in *Cepheus* and are adjacent regions.

The photograph with the 10-inch is of a great nebula found with the 6-inch Willard lens in the summer of 1893 (see *Knowledge* for January 1894). The center of this object may be taken at the star *B. D.* $+56^{\circ}.2617$, the position of which for 1855.0 is R. A. $= 21^{h}\ 34^{m}\ 29^{s}.8$; Dec. $= +56^{\circ}\ 49'.7$. The nebulosity is roughly roundish and is about 2° in diameter. It is broken up with peculiar dark rifts, not unlike those in the Trifid nebula, and involves a group of moderately bright stars. It lies along the boundary of two regions, one

rich in stars and the other comparatively poor. I have previously called attention to this peculiarity of many of these great diffused nebulosities.[1] The sky was not specially suited for photographing such an object and I hope to repeat the exposure under better conditions, and shall expect to show much more nebulosity, as greater extensions are faintly indicated. The scale of this picture is 1 inch $=0\overset{\circ}{.}84$. It is enlarged 1.36 times from the original.

The second photograph, with the 6¼-inch, shows the peculiar structure of the Milky Way in that part of the sky. The groundwork here is made up of comparatively small stars with a scattering of relatively bright ones. It is a fair sample of what can be done with the ordinary commercial lens which was never intended for this kind of work. It is very slightly enlarged from the original, and the scale is 1 inch $=1\overset{\circ}{.}8$, or about one-half that of the other photograph. Though the definition at the corners of the plate is poor, from spherical aberration, it was thought best to include these corners, since they carry out the structural details without being seriously offensive.

For many years I have called attention repeatedly to the fact that many of the nebulæ occupy vacant regions as if their existence was in some way the cause of the scarcity of stars. Perhaps the most remarkable instance of this kind is the region of the great nebula of *ρ Ophiuchi* where the nebula is situated in apparently a large hole in the Milky Way, and this hole is connected with a remarkably accentuated vacant lane running from it to the east for many degrees.[2] This peculiarity is also noticeable in the case of the great nebula of *Orion*, where the region occupied by the nebula seems to be vacant of the very small stars that form the general background of the sky in that part of the heavens. Though there are many small stars connected with the nebula, they do not appear to belong to the regular background of the sky there, and perhaps are all intimately associated with the nebula itself.

In reference to these dark lanes and holes, there seems to be a growing tendency to consider them dark masses nearer to us than the Milky Way and the nebulæ that intercept the light from these objects.

[1] See *Astrophysical Journal*, **2**, 350, December 1895.

[2] See *Popular Astronomy* for September 1897 (Vol. **5**, p. 227) for a full description of these features.

PLATE III

N.

NEBULOSITY IN THE MILKY WAY

Center at R.A. = $21^h\ 35^m$; Dec. = $+57°$. October 6, 1904, from $7^h\ 14^m$ to $12^h\ 25^m$ C. S. T.

10-inch Lens. Scale: 1 cm = $0\overset{\circ}{.}33$

This idea was originally put forward by Mr. A. C. Ranyard. Though this may in a few cases be true—for some of them look very much that way—I think they can be more readily explained on the assumption that they are real vacancies. In most cases the evidence points palpably in this direction. In the few cases where the appearance would rather suggest the other idea—and this is mostly in reference to the nebulæ—the evidence is still not very strong.

The 6¼-inch lens does not cover sharply as large a field as the Willard lens, but it must be remembered that the Willard was refigured by Brashear. In the lower left-hand corner of this picture, where the definition is poor, are seen the vacant lane and small nebula shown in Dr. Max Wolf's beautiful photograph reproduced in *Monthly Notices*, **64**, 838. The large nebulous cloud shown in the northwest part of his plate does not seem real. It does not look like nebulosity on the photograph itself, and it is not shown on either the present plate or the one with the 10-inch made at the same time. It must be a defect of some kind. I have been familiar with this vacant lane and small nebula to which Dr. Max Wolf calls attention, since October of 1893. I have a photograph clearly showing them on October 11, 1893.

The relationship of the two present photographs will be seen from the fact that in the picture with the 6¼-inch the small star 0.6 inch from the top and 2.3 inches from the right-hand side is the conspicuous star near the lower right-hand corner of the 10-inch photograph.

Through the deep interest of Professor Hale in the possibilities of the Bruce telescope, we decided, at a discussion of the subject in the past summer, temporarily to transport the telescope to the summit of Mount Wilson (6,000 ft. elevation) near Pasadena in southern California, where Professor Hale has already established a branch of the Yerkes Observatory for solar work. The telescope was therefore taken down and shipped to California on December 5. It is intended to use the lower latitude of Mount Wilson to reach those magnificent regions of the Milky Way in *Sagittarius* and *Scorpio* which are not attainable from the latitude of the Yerkes Observatory, and to secure photographs of them during the coming summer. It is also hoped to utilize the transparent sky of Mount Wilson to photograph some of the great diffused nebulosities that are more or less cut out by the denser air at lower altitudes.

The $6\frac{1}{4}$-inch doublet is soon to give place to a new 6-inch doublet made from the new Jena ultra-violet glass, which is specially suited for photographing the nebulæ. It is expected that this will be finished in time to be used at Mount Wilson. An objective-prism of the same glass and of equal aperture has also been ordered from Zeiss & Co.

The telescope has been in use since the middle of April, until it was taken down in the last of November for shipment to California. In that time some eighty negatives were obtained with each camera, though the season has been unfavorable for work of this kind. One striking feature of these photographs is the number of meteor trails that have been obtained. Nearly a dozen have been photographed. On two different dates two trails were obtained on the plates. The first of these were two bright Lyrids that followed each other closely on April 20. The two trails are almost in a straight line—one coming on the north side of the plate and the other going off to the south. At first glance it looks like one meteor trail interrupted in the middle. On November 15 a fine Leonid trail in *Orion* was obtained. This same meteor was photographed by Mr. Sullivan with a 4-inch camera attached to the end of the 40-inch telescope during the spectrographic observations by Professor Frost. Though the distance between the two instruments was only about 400 feet, there is a decided parallax, from which the distance of the meteor will be obtained.

In conclusion, I wish to express my very deep obligations to Professor Hale for the courtesy and kindness he has constantly shown me throughout all the wearisome time it has taken to get into working existence this valuable gift of Miss Bruce. I am also greatly indebted to Professor Frost for a continuation of these same good services. To Messrs. Warner and Swasey I am most deeply indebted for the interest they have taken in the making of this telescope, and for the fact that the price paid for the instrument has been a secondary consideration with them. They have given more than they have been paid for, and have taken a genuine pride in the work from its beginning. I am indebted to Dr. Brashear for his earnest endeavors to make a lens to fulfil all the requirements that were demanded of it, and for the excellence of the lens he finally produced.

YERKES OBSERVATORY,
December 1904.

PLATE IV

Milky Way in Cepheus

Center at R.A. = $21^h\ 30^m$; Dec. = $+\ 49^\circ.5$. September 11, 1904, from $8^h\ 0^m$ to $14^h\ 40^m$ C. S. T.

$6\frac{1}{4}$-inch Lens. Scale: 1 cm = $0^\circ.7$

THE SPECTROHELIOGRAPH OF THE POTSDAM OBSERVATORY

By P. KEMPF

Interest in the photography of the Sun in monochromatic light has been decidedly increased by the important results obtained by Hale and Ellerman[1] with the Rumford spectroheliograph, designed by them and used in connection with the forty-inch refractor of the Yerkes Observatory; and it is scarcely to be doubted that the efforts of the American astronomers to enlist more observers in these investigations than have hitherto devoted themselves to them will be successful.

For a number of years the writer has made regular observations with the spectroheliograph—at the Astrophysical Observatory of Potsdam, accounts of which have been given in the annual reports of the observatory in the *Vierteljahrsschrift der Astronomischen Gesellschaft.* A detailed communication as to these observations has, indeed, not yet been made, as it was my purpose to publish the results at the same time. Preoccupation with other work has hitherto frustrated this purpose, but I would now give at least a description of the apparatus and method of observation, since it is presumable that more such instruments will be used in the near future, and hence the description of the experience with the instrument might be of value.

The spectroheliograph used by me is essentially of the form used by Hale[2] on Mount Etna in his attempts to photograph the solar corona. In the first instrument built by Hale the two tubes of the spectrograph formed an angle of 25° with each other. They were rigidly attached to the refractor, and the two slits only were moved by a system of levers, simultaneously and in opposite directions.[3] This form had, as is well known, the disadvantage that the two slits must be given unequal velocities, so that the resulting photographic image was not round, but ellipitical. This defect was avoided in the

[1] *Publications of the Yerkes Observatory*, Vol. **3**, Part I, 1903.

[2] *Astronomy and Astro-Physics*, **13**, 681, 1894.

[3] *Ibid.*, **11**, 407, 1892.

second form, that designed for Mount Etna, by placing the collimator and camera tubes parallel and moving the whole spectroscopic apparatus past the fixed focal image and photographic plate. This form is doubtless the simplest, and, for instruments not too large, the most advantageous, and I chose it on that account for the Potsdam apparatus.

The Potsdam apparatus was constructed by Otto Töpfer and Son in Potsdam, who built the Etna instrument for Hale. Plate V gives a general view of the apparatus, Fig. 1, a drawing of the cross-section, and Plate VI shows the instrument attached to the Grubb refractor of the Potsdam Observatory.

Fig. 1 shows the two adjacent parallel tubes of the spectrograph, the collimator C and the camera K, with the objectives O_1 and O_2 and the slits S_1 and S_2. The tubes are attached at the objective and slit ends to stiff metal plates which form the support for the movable part of the apparatus. These plates are bound together by two strong T-shaped iron bars T, T. The objectives project into an aluminum box which contains the grating G, together with a mirror and a total-reflection prism. These are so adjusted that the beam of light falling on the grating makes an angle of 30° with the optical axis of the camera tube. The two telescopes are also made of aluminum. The tube K is divided at t, and the whole slit part may be removed and be replaced by a much wider conical piece which makes it possible to photograph a longer portion of the spectrum. The junction T is made light-tight by a metal collar.

Objectives and slits are separately movable in the direction of the optical axis. The width of the slit S_1 may be regulated by a micrometer screw which moves one jaw of the slit. On the other hand, the jaws of S_2 open symmetrically, so that the middle of the slit remains in the same place. Furthermore, the whole slit-plate can be moved at S_2 perpendicularly to the optical axis, in order that the slit may be conveniently set upon a given spectrum line. The change of slit-width as well as the movement of the whole slit is effected by keys which can turn the screw heads s, Fig. 1, from outside. The amount of the displacement is read on the ocular scale of a microscope which also, with the help of a total reflection prism, enables the observation of the spectrum through the second slit (Plate V).

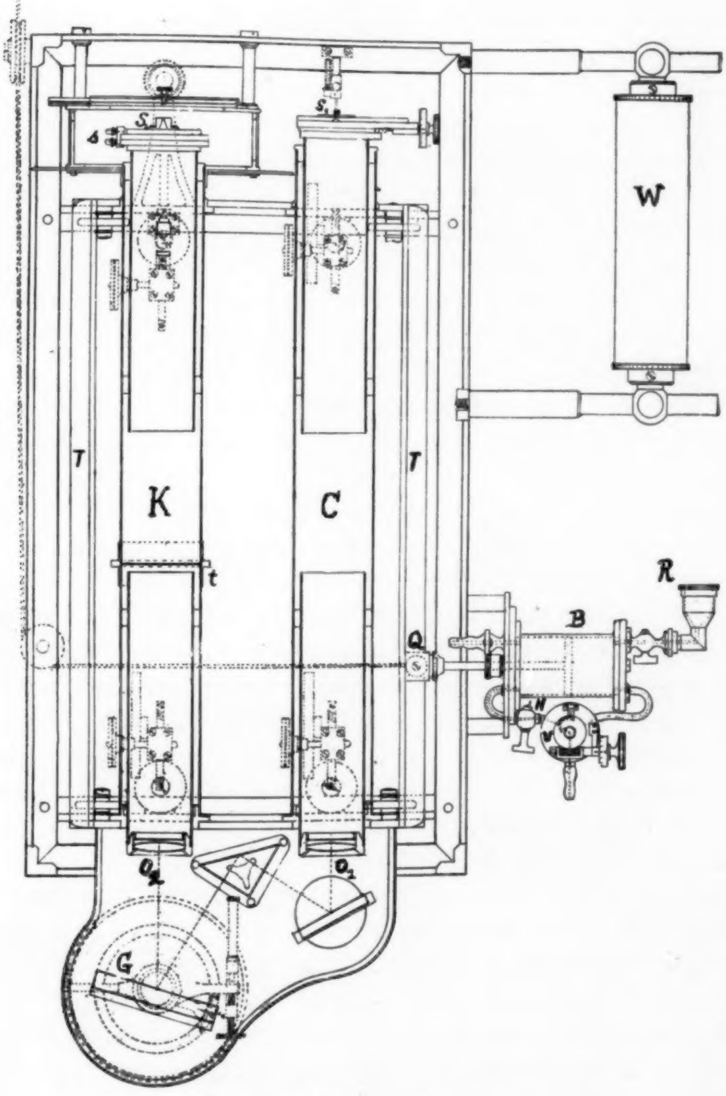


FIG. 1.—Plan of Spectroheliograph.

The motion of the apparatus is accomplished in the following manner: Four right-angle ways are attached to the box-shaped iron frame which incloses the whole apparatus. On these ways run sixteen rollers, which are attached in pairs perpendicular to each

other, to the above-mentioned metal plates (which carry the objectives and slits). The driving is accomplished by a weight which is attached at the two ends of a small cross-piece *Q* resting upon one of the T-shaped connecting pieces (Fig. 1).

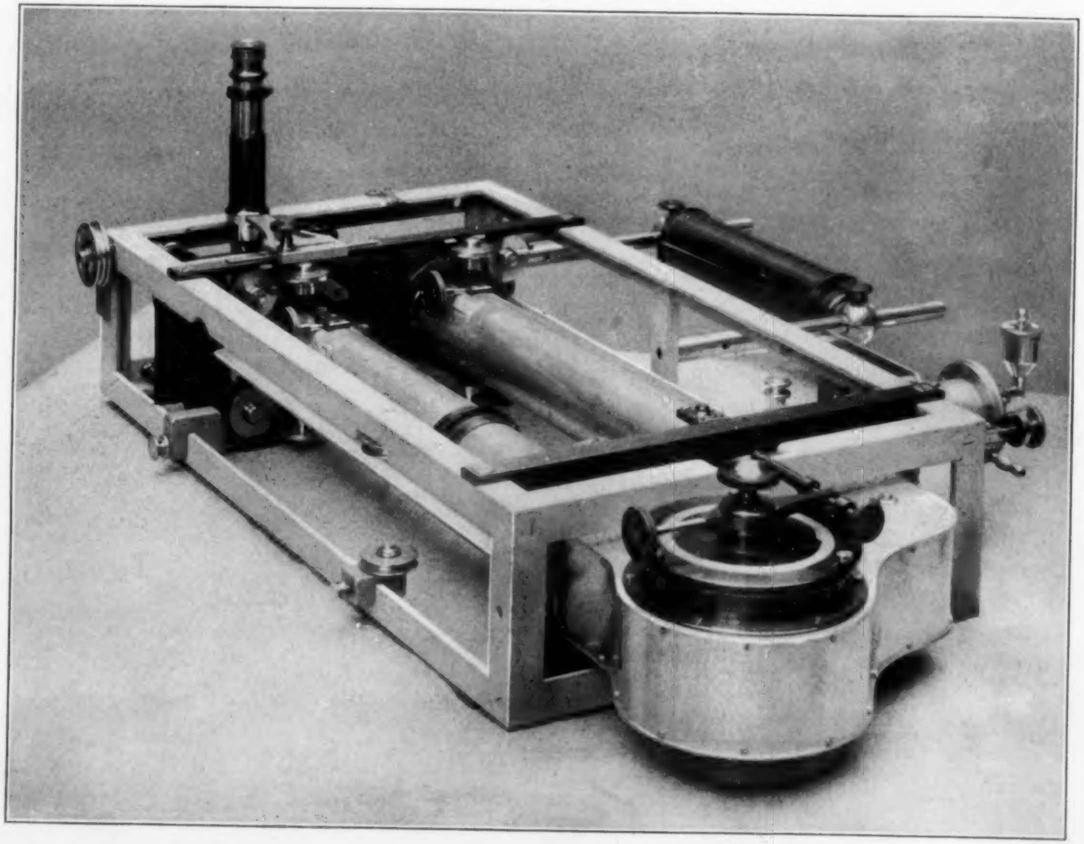
Two cords go from the bar *Q* over two rollers to a little pulley, and from there as a single cord over a pulley mounted on an extension of the declination axis to a weight (see Plate VI), which may be increased as desired by adding disks.

The motion is regulated by a clepsydra *B*, whose piston is attached to the same place as the driving-weight. The tube connecting the two ends of the cylinder is interrupted in two places—at *H* by an ordinary cock and at *v* by a micrometric cock capable of varying the outflow between wide limits. In order to prevent air from entering the regulator, I had a reservoir attached at *R* which is kept constantly filled with liquid. While the instrument is not in use the valves are opened so that the water may have free circulation, the drain of water from the reservoir being immediately resupplied. In filling the reservoir Hale's plan of using a mixture of water and 5 per cent. of glycerin was followed, as such a mixture resists severe cold without freezing.

The whole moving mechanism works very smoothly and is wholly free from vibration, so that it may be highly recommended, especially for small instruments.

The manner of attaching the apparatus to the refractor is shown in Plate VI. The collimator lies in the prolongation of the optical axis of the telescope, and in order to balance the camera tube, etc., the counter weight *W* (see Fig. 1) had to be attached. The disk to which the frame is attached may be rotated in position angle so that the apparatus can be moved in any direction desired. It is most advantageous to keep this motion parallel to the diurnal motion, and then to keep the apparatus unchanged in this position, the deviations from this adjustment being determined at appropriate intervals. The direction of motion of the apparatus is marked on the photograph by the lines traced by small dust particles on the first slit. To provide against the desirable case where all dust particles are removed from the slit, a thin metallic wire is run across the first slit, and it appears on the photograph (Sun's image) as a sharp and easily measurable line. This wire serves also to deter-

PLATE V



The Potsdam Spectroheliograph

mine the error of position angle. The second slit for this purpose is opened as wide as possible (in my instrument 3 to $3\frac{1}{2}$ mm) and a photograph of the spectrum is made. Then the apparatus is run a considerable distance, and the driving-clock of the refractor is stopped so that the solar image also moves forward. At the instant when the Sun is again centrally on the slit a second photograph of the spectrum is taken. Measurement of the distance in declination of the wire from the center of the Sun on the two photographs then gives the deviation of the motion of the apparatus from the direction of the diurnal motion.

As previously mentioned, the spectrum is produced by a plane grating, furnished by Brashear, with a ruled surface of 2×3 inches, and about 14,000 lines to the inch. It is undeniable that for the purpose in hand prisms have advantages in many respects, especially as they give less diffuse light than the grating. I was not in position to employ prisms, however, as my eyes are not sufficiently sensitive to violet light to be able to bring the K line with certainty into the slit of only about 0.1 mm width. With the grating the K line in the fourth order, which I use for my observations, coincides with the green of the third order, so that the setting may be made upon a green line, the distance of which from the violet line has been determined once for all.

As to the diffuse light, the camera tube is further furnished with a system of diaphragms which grow constantly smaller from the objective to the slit, keeping diffuse light off the second slit as far as possible. Similarly a diaphragm is placed between the grating and the total-reflection prism. It divides the whole space into two parts and allows the passage of only the light coming through the prism.

The Grubb refractor, to which the spectroheliograph is attached, has a focal length of 3.2 m, and the focal image of the Sun is therefore about 30 mm in diameter. The following dimensions were accordingly chosen for the various parts of the spectroheliograph. The length of the slits and the clear aperture of the objectives is 45 mm, the focal length[1] of the collimator and camera 600 mm. The cylinder of the regulator is 5×8 cm and permits a total motion of the apparatus

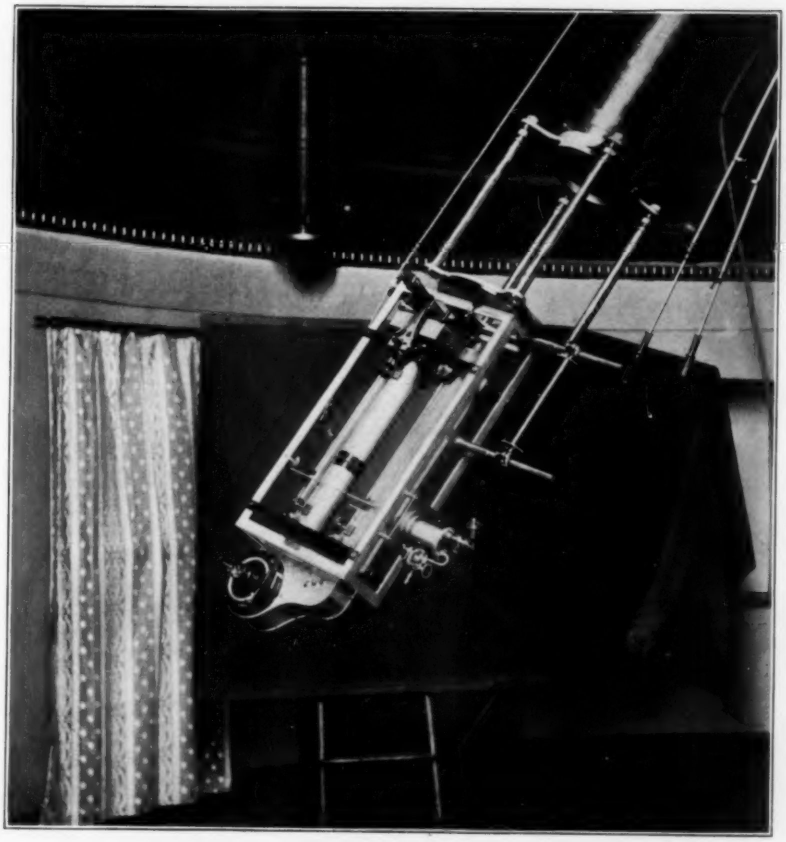
[1] The focal length might better have been smaller.

of about 6 cm. The frame which incloses the whole apparatus measures $68 \times 36 \times 13\frac{1}{2}$ cm.

It is clear that an instrument of these modest dimensions cannot give results comparable to those of the Rumford spectroheliograph at the Yerkes Observatory in which the solar image is 18 cm in diameter. In particular it is not possible to bring out the fine structure of the Sun's surface, which the beautiful photographs of Messrs. Hale and Ellerman reveal. All coarser structure may be perceived on our photographs, however, and can be measured under low magnification with adequate accuracy. Unfortunately, it is not probable that any considerable number of instruments with the great dimensions of the Rumford spectroheliograph will be built, so that it is the more to be desired that at least there shall come into use as many instruments as possible of the dimensions described here, which do not involve extraordinary expense.

POTSDAM, ASTROPHYSICAL OBSERVATORY,
October 1904.

PLATE VI



SPECTROHELIOGRAPH ATTACHED TO REFRACTOR

ON THE DETERMINATION OF RADIAL VELOCITIES AT POULKOVA

By A. BÉLOPOLSKY

In my last article on "Standard Velocity Stars,"[1] I mentioned among other things an error, the cause of which was at the time not clear to me. It was the fact that the light of a star, projected on the slit of the spectrograph by the thirty-inch objective, did not wholly fill the objective of the collimator, but only illuminated one-half of it.

Another error which I was unable to overcome for a long time was that the lines of the stellar spectrum always appeared inclined with respect to the artificial spectrum and, probably for this reason, were never as sharp as the appearance of the artificial lines would lead one to expect.

I first sought the cause of these errors in the spectrograph itself, of which I will now give a few details. The instrument is essentially a copy of Spectrograph III of the Potsdam Observatory. The length of the collimator is 614.7 mm for the setting 10.0; that of the one camera is 605.3 mm (always reckoned from the front surface of the lens) at the setting 13.2; and of the other camera, 413.1 mm at the setting 18. The rays of the spectrum are united as follows by the long camera (A) as determined by Hartmann's method:

Edge of Wedge under Plate Points toward Violet				Edge of Wedge under Plate Points toward Blue			
λ	Setting	λ	Setting	λ	Setting	λ	Setting
400	9.2	430*	13.2	406	7.8	438	13.1
410	12.0	438	12.5	414	9.4	444	13.1
414	12.8	444	11.9	420	10.6	450	13.0
420	13.9	450	11.3	430	12.6	454	13.0
426	13.8	454	10.9	434	13.1	...	....

T. = −0°.5 C. T. = −4°.0 C.

The camera setting for wave-lengths between λ 414 and λ 440 changes with the temperature as follows:

[1] *Astrophysical Journal*, **19**, 85, 1904.

T.	Setting	T.	Setting
+0° C.	13.0	+12° C.	12.4
4	13.0	14	12.0
8	13.0	16	11.4
10	12.8	..	....

For the short camera (B) we obtain the following camera settings for different wave-lengths. The first table is for the position with the line $H\gamma$ at the center of the field, and the settings remain unchanged for the range of temperature from $-2^\circ.4$ to $+11^\circ.5$ C. The settings are also given for the cases where the other hydrogen lines are at the center of the field.

$H\gamma$ Central		$H\beta$ Central		$H\delta$ Central		$H\epsilon$ Central	
λ	Setting	λ	Setting	λ	Setting	λ	Setting
406	17.3	440	15.8	396	18.5	384	14.9
410	17.4	444	16.3	400	18.5	388	15.6
414	17.6	448	16.7	404	18.5	392	16.7
418	17.7	452	17.2	408	18.4	396	17.8
422	17.7	456	17.6	412	18.2	400	18.5
426	17.8	460	17.9	416	17.7	...	....
430	17.9	464	18.0	420	17.2	...	....
434	18.0	468	18.1	424	16.6	...	....
438	17.9	472	18.1	428	16.3	...	....
442	17.8	476	18.1	432	15.0	...	....
446	17.7	480	18.1	...	....	...	....
450	17.6	484	18.1	...	....	...	....
454	17.4	488	18.0	...	....	...	....
...	....	492	17.8	...	....	...	....

All this holds for the particular setting of the collimator at 9.8. It is possible to obtain a better focusing of the different rays in a surface for different settings of the collimator scale. For instance, with the collimator setting of 12.0, I obtained the following settings on the scale of camera A:

λ	Setting	λ	Setting
426	14.1	446	14.5
428	14.4	450	14.5
430	14.5	454	14.5
438	14.5	458	13.8
442	14.5	...	....

The edge of the wedge was pointed toward the blue end of the spectrum, and the temperature was $+16°$ C.

For this spectrograph, the curvature of the lines was determined in the following way: Two plates of the iron spectrum were taken with as long a slit as possible, and one spectrum was laid upon the other, under the microscope of the measuring machine, in such a way that the curvature was opposite in the two cases. Identical lines were then brought toward each other until their tips coincided. This gave a figure like a convex lens, the measurement of the thickness of which yielded double the correction for the reduction of the tips of the lines to the center. The length of the line being also known, the parameter of the parabola is computed from these two data. Denoting by x the correction for the curvature, or the abscissa of the parabola, by $2y$ the double ordinate (length of the chord of the "lens"), and by p the parameter, we obtain for the two cameras the following values:

A, $2y=11.8$ mm		B, $2y=8.1$ mm	
λ	$2p$	λ	$2p$
4102	6.16	4102	3.47
4261	6.53	4155	3.47
4427	7.39	4261	3.86
....	....	4315	4.07
....	....	4427	4.19
....	....	4510	4.21

Thence we obtain by means of Hartmann's dispersion formula the following table of corrections, x, together with their values in kilometers:

λ	Camera III A				Camera III B			
	$y=1$ rev.		$y=2$ rev.		$y=1$ rev.		$y=2$ rev.	
	x	x	x	x	x	x	x	x
418........	0.16 p	0.19 km	0.63 p	0.76 km	0.28 p	0.48 km	1.10 p	1.98 km
422........	.16	.21	.63	.82	.27	.52	1.07	2.06
426........	.15	.21	.61	.85	.26	.52	1.03	2.11
430........	.15	.23	.60	.90	.25	.54	1.00	2.17
434........	.15	.24	.59	.90	.25	.57	0.98	2.21
438........	.14	.24	.57	.91	.24	.57	.97	2.33
442........	.14	.24	.55	.93	.24	.61	.96	2.42
446........					.24	.63	.96	2.51
450........					.24	.65	.95	2.59
454........					.24	.69	.94	2.70

The collimator was very accurately set in line with the optical axis of the thirty-inch telescope. An electric thermostat was also used with exposures on stars, which kept the temperature of the prisms constant for several hours within a range of 0°.5 C.

Nevertheless, in spite of all precautions, the error mentioned at the beginning of this article could not be overcome. The measurement of a series of stellar spectrograms taken under different conditions showed that the difference of the settings on the edges of the spectrograms always kept the same sign, while the absolute magnitude varied. For instance, when the spectrogram was laid on the microscope stage with film up, the settings on the upper edge were always larger than on the lower edge. Only in the position with the camera upward did these differences become smaller and, in a few instances, change their sign. All the other changes were ineffectual on these plates—change of slit-width, of temperature, of width of spectrogram, of hour-angle, and of the diaphragms placed over the collimator lens. (The diaphragms were round and also of the form of segments with the chord parallel to the edge of the prism.) The following table gives these differences arranged for each plate according to the wave-length after a graphical adjustment. If we take the mean for each plate, and then unite these for the mean values for the position of the camera above and camera below, we obtain

For camera below, +0.019 rev.
For camera above, −0.001 rev.

It appears that the position of the camera has an effect on the sign of the differences. If we disregard a progressive change in the values depending upon the wave-length, probably due to the diffuseness of the lines at the edge of the field of view, they seem to be otherwise independent of all of the circumstances, such as temperature, camera, width of spectrogram, etc. The measurements of the spectrograms of the Moon show on each plate a progressive change of the differences dependent on the wave-length, but in the mean they are practically zero (−0.0015). It would, therefore, seem that the error under discussion does not occur for spectrograms of disks.

The dependence of the magnitude of the error upon the position of the camera led me to give attention to the optical parts outside of

the spectrograph. It is well known that the thirty-inch telescope has a correcting lens for the violet rays when used for spectrographic purposes. This lens is attached to the tube by a side arm, and is placed at a distance of 174 mm from the focal plane of the thirty-inch objective. This arrangement renders very difficult the centering of this lens; and since its diameter is 60 mm, while the diameter of the cone of rays is 50 mm, it is possible that on account of the flexure of the tube a portion of the rays for certain hour-angles and declinations passes to the side of the lens and unites on the slit of the spectrograph as a visual star. This is seen in the guiding telescope of the spectrograph and held upon the slit. But it may be that this visual image does not coincide with the photographic image. In order to convince myself that this did not happen in our case, the instrument-maker of the Observatory, Mr. Freiberg, made at my request a second guiding telescope for the spectrograph, which was planned to utilize the rays from the first surface of the second prism, hence to see the spectrum of the star. In this way it was possible, without taking the instrument apart, to observe at the same time the direct image of the star and also its spectrum.

The peculiarity at once appeared that when the visually brightest part of the star was set upon the slit, the spectrum between λ 430 and λ 450 was not to be seen; while if the instrument was so held that the spectrum appeared bright, the star itself disappeared from the slit. By means of a micrometer attached to the forty-foot finder of the thirty-inch telescope I then measured the difference in position of the entire instrument for the case of bright star and bright spectrum, and found in the mean a difference of $2''.5$ to $3''$.

I then undertook to make a series of spectrograms by first guiding upon the star and then upon the spectrum. I employed for the purpose *α Boötis*, and obtained in all eight pairs of plates, the measurements of which I give below.

Measurements were made in both directions, and the differences of the setting on the edges of the spectrogram were always formed in the sense upper edge minus lower edge, as seen in the microscope.

TABLE I

DIFFERENCES, IN REVOLUTIONS OF THE SCREW, BETWEEN SETTINGS ON THE UPPER AND LOWER EDGES OF STELLAR SPECTROGRAMS

CAMERA B 1903	*α Persei*					*γ Cephei*	*δ Cephei*		
	Oct. 26	Dec. 12	Dec. 20	Dec. 30	Dec. 30	Sept. 8	Sept. 21	Sept. 18	Sept. 17
Slit-width	18	20	18	15	21	20	20	20	20
Camera setting	18.5	18	18	18	18	18	18	18	18
Temperature	$+1\overset{\circ}{.}5$	-5°	-6°	-6°	-6°	$+12^\circ$	$+13^\circ$	$+12^\circ$	$+11^\circ$
Hour angle	$2^h\ 5^mE$	$2^h\ 20^mE$	$2^h\ 15^mE$	$2^h\ 12^mE$	$1^h\ 6^mE$	$0^h\ 35^mE$	$2^h\ 24^mE$	$2^h\ 24^mE$	$2^h\ 27^mE$
Position of camera	below	below	below	below	below	below	below	above	above
Width of spectrogram	57	84	84	89	89	72	81	81	81

DIFFERENCES: UPPER EDGE—LOWER EDGE

λ 420	....	....	+0.025	+0.030	+0.010	+0.004	....	0.000	....
425	+0.008	+0.032	+ .012	.016	0.017	....	−0.003	.000	−0.002
430	.008	.020	+ .002	.008	.020	.007	+ .003	+ .009	− .002
435	.008	.018	− .001	.007	.018	.011	+ .012	+ .002	− .002
440	.009	.021	− .003	.008	.016	.013	+ .017	− .002	− .003
445	.010	.050	− .002	.015	.010	.020	+ .016	+ .006	− .002
450	.010	....	.000	.035	....	....	+ .012	+ .013	+ .003
Mean	+0.009	+0.030	+0.005	+0.017	+0.015	+0.011	+0.010	+0.004	−0.001

The reading of the scale on the draw-tube was 45 on each day except the first, when it was 40. On the second and third dates a circular diaphragm of 28 mm aperture was used; otherwise none was used.

CAMERA A 1903	*α Boötis*					*β Geminorum*	*α Boötis*		
	Apr. 21	May 8	May 16	May 9	May 17	May 2	1904 May 12	May 17	May 20
Slit-width	16	17	17	16	16	19	13	13	13
Camera setting	13.1	13	13.6	13	13.5	13	13.5	13.2	13.2
Temperature	$+4^\circ$	$+5\overset{\circ}{.}5$	$+8^\circ$	$+2\overset{\circ}{.}8$	$+7^\circ$	$+1^\circ$	$+6^\circ$	$+8^\circ$	....
Hour angle	$0^h\ 45^mE$	$0^h\ 28^mE$	$0^h\ 29^mE$	$0^h\ 15^mE$	$0^h\ 7^mE$	$3^h\ 9^mW$	$1^h\ 2^mE$	$1^h\ 16^mW$	$0^h\ 47^mW$
Position of camera	below	below	above	above	above	below	below	below	below
Width of spectrogram	270	270	270	270	270	130	160	360	160

TABLE I—*Continued*

DIFFERENCES: UPPER EDGE—LOWER EDGE

λ 420	....	....	....	....	−0.010	+0.002	....	....	....
425	+0.020	0.000	−0.010	−0.005	− .006	.012	....	....	....
430	.010	.000	− .013	− .003	− .001	.020	+0.039	+0.032	+0.031
435	.006	.000	− .009	.000	+ .008	.030	.022	.033	.043
440	.010	− .001	.000	.000	+(.017)	.039	.023	.037	.053
445	.010	+ .001	.000	.000	+(.018)	.045	.032	.035	.045
450	.010	+ .004	....	+ .004	+ .019	....	.028	.040	.037
Mean	+0.011	+0.001	−0.006	−0.001	+0.006	+0.025	+0.029	+0.039	+0.042

The reading of the scale on the draw-tube was 40 on each date. On the second, third, fourth, and fifth plates the diaphragm in form of a segment was used.

SPECTROGRAMS OF MOON

1904	Feb. 26	Feb. 28	March 25 (1)	March 25 (2)
λ 418	−0.018 rev.	....	....	....
420	− .005	....	....	....
422	+ .022	....	....	....
424	− .005	+0.004	....	....
425	− .006	....	....	....
427	000	....	.....	....
428	....	+ .019	0.000	−0.007
429	....	....	....	− .016
430	....	....	....	− .010
431	− .007	....	− .003	+ .002
432	....	+ .007	....	....
434	+ .009	....	....	+ .004
435	− .016	+ .009	.000	+ .012
437	....	....	....	+ .023
440	....	....	− .004	+ .024
444	....	....	....	+ .016
445	....	....	− .004	....
447	....	− .008	....	+ .017
449	....	....	....	+ .009
450	....	....	+ .004	....
453	....	− .003	....	+ .007

				λ 428–433	λ 435–453
Mean	−0.003 rev.	+0.005	−0.001	−0.005	+0.015
Orientation in microscope	+0.004	−0.010	+0.003	−0.006	−0.016
Inclination to comparison lines	+0.001 rev.	−0.005	+0.002	−0.011	−0.001

TABLE II

DIFFERENCES IN SETTINGS ON UPPER AND LOWER EDGES OF

Date Guided on	May 12 Star	May 13 (1) Spectrum	May 13 (2) Spectrum	May 17 (1) Spectrum	May 17 (2) Star	May 20 (1) Star	May 20 (2) Spectrum	May 25 (1) Spectrum	May 25 (2) Star
λ									
4294	+0.044	0.000	+0.013	......	+0.030	+0.026	......	......	+0.030
4308	......	......	......	......	......	......	......	......	......
4315	.037	+ .001	+ .001	+0.001	.035	.036	+0.010	+0.008	.014
4325	......	+ .003	......	+ .012	......	......	......	+ .007	......
4326	......	......	......	......	......	.033	......	......	......
4335	......	......	......	......	......	......	......	+ .006	......
4337	0.37	− .005	− .008	− .018	......	......	+ .016	......	......
4340	......	......	......	......	......	......	......	......	......
4345	......	......	− .012	+ .002	......	......	......	......	......
4348	......	......	......	− .011	......	......	......	......	......
4352	......	......	......	......	......	......	− .006	......	......
4353	.016	− .006	+.011	− .002	.040	.049	+ .004	− .002	.019
4356	......	......	......	......	......	.038	......	......	......
4359	......	......	......	......	......	......	+ .008	......	.035
4360	......	......	......	......	......	......	......	......	......
4366	......	......	......	......	.030	......	......	......	......
4371	......	......	......	......	......	......	......	......	.046
4372	......	− .001	......	......	......	......	......	......	......
4376	......	......	+ .005	− .005	......	......	......	+ .007	.032
4379	......	+ .001	− .004	......	......	......	......	......	......
4401	......	+ .017	......	......	......	......	......	......	......
4405	.023	+ .007	− .006	......	.037	......	......	......	......
4406	......	......	......	......	......	......	......	+ .006	......
4407	......	......	− .004	+ .005	......	......	......	+ .002	......
4408	......	......	+ .005	......	......	......	......	......	......
4409	......	+ .010	......	......	.040	.033	+ .003	......	.028
4415	......	+ .003	......	......	......	......	......	......	......
4427	.020	+ .003	− .002	− .006	.046	.042	+ .006	+ .005	.034
4442	......	......	......	......	......	......	......	......	......
4443	......	+ .001	......	− .003	......	......	......	......	......
4448	.033	......	......	......	......	......	......	− .006	......
4451	......	......	......	......	......	......	......	......	......
4459	.030	+ .001	− .008	......	......	.052	+ .020	+ .005	.030
4460	......	......	......	......	......	......	......	......	......
4461	......	......	......	......	......	.037	......	......	......
4467	......	......	......	......	.036	......	......	......	.025
4476	......	+ .001	......	......	.054	......	......	......	......
4482	......	......	− .002	......	......	......	+ .015	......	.034
4485	......	......	......	......	......	......	......	......	......
4495	......	+ .004	......	......	......	.021	......	......	......
4496	......	......	......	......	......	......	......	......	......
4527	......	......	......	......	......	.052	......	......	......
4529	.028	− .002	− .004	......	.039	......	+ .031	+ .012	.046
4531	......	......	......	......	......	......	......	......	......
4536	......	+ .003	......	......	.043	.045	+ .022	+ .009	.043
4550	......	......	− .003	......	......	.038	......	+ .009	.047
4566	......	......	......	......	......	......	......	......	......
4600	......	......	− .001	......	......	......	......	......	......
4603	......	......	......	......	.051	......	......	......	......
Means	+0.030	+0.002	−0.001	−0.002	+0.040	+0.040	+0.012	+0.005	+0.033

TABLE II

Spectrograms of *a Boötis*, 1904, Expressed in Revolutions of the Screw

Date Guided on	May 25 (3) Spectrum	May 26 (1) Star	May 26 (2) Spectrum	May 29 (1) Spectrum	May 29 (2) Star	May 30 (1) Star	May 30 (2) Spectrum	May 31 (1) Star	May 31 (2) Spectrum
λ									
4294	+0.015	+0.030	+0.030	......	+0.030	+0.055	......	......	......
4308	......	......	......	......	......	......	......	......	+0.011
4315	−0.003	.006	.018	+0.006	.024	.022	+0.014	+0.026	(+ .022)
4325	− .013	......	.013	......	......	......	.004	......	......
4326	......	......	......	.006	......	.032	......	......	+ .004
4335	......	......	......	......	......	......	.001	......	......
4337	......	......	.029	.012	......	......	......	......	......
4340	......	......	.001	......	......	.043	......	......	......
4345	+ .005	......	......	......	......	......	......	.041	......
4348	......	......	......	......	......	......	......	......	......
4352	......	......	......	......	......	.033	......	......	......
4353	+ .002	.023	.001	.014	.029	......	.013	.020	+ .002
4356	......	......	......	......	......	......	......	......	......
4359	......	......	......	......	......	......	......	......	......
4360	+ .009	......	......	......	.030	......	......	......	......
4366	......	......	......	......	......	......	......	......	......
4371	......	......	......	......	.033	......	......	......	......
4372	......	......	......	......	......	......	......	......	......
4376	......	......	......	.008	......	.039	.010	......	......
4379	......	......	......	......	......	......	......	......	− .008
4401	......	......	......	......	......	......	......	......	......
4405	......	......	......	......	......	......	......	......	......
4406	......	......	......	......	......	......	......	......	+ .005
4407	......	......	......	......	......	......	......	......	......
4408	......	.040	......	......	......	......	......	......	......
4409	+ .017	......	......	......	.029	.028	.005	.020	......
4415	......	......	......	.008	......	......	.004	.034	......
4427	+ .012	.020	.017	.016	.028	.028	.006	.026	+ .007
4442	......	......	......	.002	......	......	......	......	......
4443	......	......	......	......	......	......	......	......	......
4448	......	......	.016	......	......	......	......	......	......
4451	......	......	......	......	......	......	......	.036	......
4459	+ .003	.021	.021	......	.044	.030	......	.026	+ .010
4460	......	......	......	......	......	......	.005	......	......
4461	......	......	......	......	......	......	......	......	......
4467	......	......	......	.005	......	......	......	......	......
4476	......	.030	......	.015	......	......	......	.020	......
4482	+ .005	......	.013	......	.039	......	.008	......	......
4485	......	......	......	......	......	......	......	......	.000
4495	......	......	......	......	......	......	......	......	......
4496	+ .007	......	.011	......	......	......	......	.023	......
4527	......	......	......	......	......	......	......	......	......
4529	+ .016	.032	.009	.011	.032	.035	.008	.024	+ .013
4531	+ .013	......	......	......	......	......	......	......	......
4536	......	.020	......	.006	.014	.041	......	......	+ .001
4550	......	......	+ .005	......	......	......	......	......	......
4566	+ .006	......	− .004	......	......	......	......	......	......
4600	......	......	......	......	......	......	......	......	......
4603	......	......	......	......	......	......	......	......	......
Means	+0.007	+0.025	+0.013	+0.009	+0.030	+0.035	+0.006	+0.027	+0.006

If we form the means of these differences for each plate, and combine them into two groups according to whether the guiding was done by the star image or by the spectrum, we obtain the following results:

Guiding on Star				Guiding on Spectrum			
	rev.		rev.		rev.		rev.
May 12	+0.030	May 26.1	+0.025	May 13.1	+0.002	May 25.3	+0.007
17.2	040	29.2	.030	13.2	− .001	26.2	.006
20.1	.040	30.1	.035	17.1	− .002	29.1	.009
25.2	.033	31.1	.027	20.2	+ .012	30.2	.006
				25.1	+ .005	31.2	.006

If we now form the mean from each of the series, we obtain for the case of guiding by the star the difference +0.033 rev. ±0.002 (mean error always given here); and for the case of guiding on the spectrum +0.005 ±0.001. Expressed in kilometers, these differences amount to

$$+5.70\,\text{km} \pm 0.34\ (\lambda=4419)\ .$$
$$+0.85\,\text{km} \pm 0.27\ (\lambda=4409)\ .$$

We see from this how easy it is to make an error of 1 and more kilometers if, during the measurement, the settings are not made rigorously in the center of the spectrum, for the case that the spectrogram was obtained by holding the star on the slit during the guiding. The residual value of +0.85 km probably depends on the fact that my eye is blind to the violet end of the spectrum, and I can see the continuous spectrum only to $H\gamma$, so that I can give attention only to the region from $H\gamma$ to $H\beta$. The quality of the images may also have had some effect here.

Inasmuch as *α Boötis* has been repeatedly observed at several observatories, it was interesting to compare the results of my present determinations of radial velocity with the mean value of other determinations. For this purpose I computed the wave-lengths of the edges of the spectrogram and reduced them to the center by means of the differences obtained above. Since the wedge under the plate was in three instances placed with its edge toward the blue end of the spectrum, I computed a new formula. The coefficients of the

formula for $a=\frac{1}{2}$ were computed from the mean settings n on all spectrograms of the following iron lines, after the plates had all been reduced to the same dispersion.

λ 4293.410 - - - - - - $n=$ 2.579 rev.
4404.928 - - - - - - 48.565
4528.798 - - - - - - 94.306

The formula for determining the wave-length is

$$\lambda=3371.316+\left(\frac{[3.9269484]}{879.887-n}\right)^2 . \tag{1}$$

For the determination of n, it is

$$879.887-n=\frac{[3.9269484]}{(\lambda-3371.316)^{\frac{1}{2}}} \tag{2}$$

In order to judge how far this formula satisfies the measurements of other lines, I computed by formula (1) the wave-length from the mean n, and compared it with the values taken from Rowland's table.

λ	C.-O.	λ	C.-O.	λ	C.-O.	λ	C.-O.
	t.-m.		t.-m.		t.-m.		t.-m.
4294.301	+0.007	4352.908	−0.007	4427.482	+0.003	(4476.185)	....
4299.410	0	4376.105	0	4442.510	− 6	(4482.338)	....
4308.081	+ 7	4383.720	+ 5	4447.892	− 11	4494.738	+0.003
4315.262	− 4	4404.928	0	4459.301	+ 5	4528.798	0
4325.939	− 2	4415.293	+ 7	4466.727	− 2	(4603.126)	....
4337.216	− 8						

In order to compute the wave-lengths of the star lines, Rowland's values were employed, and the corresponding n was calculated by formula (2). The values of n corresponding to the *Fe* lines on each plate were reduced to these values and were graphically adjusted, and then similarly the values of n for the star lines. Finally, the wave-lengths were computed according to formula (1) and compared with Rowland's. The differences of wave-lengths were expressed in kilometers by the formula

$$S=\frac{300\,000}{\lambda}\cdot\frac{(\lambda-3371.3)^{1.5}}{[3.6253]} .$$

In the following tables, III, *a*, *b*, all the data for the determination of velocity are given. All the columns may be readily understood. V_a, V_d, and C denote the reduction to the Sun, the correction for the daily motion, and the correction for the curvature of the lines.

TABLE III*a*

α *Boötis* 1904

Poulkova M. T.	Exposure	Hour Angle	Slit-Width	Comparison Spectrum	Guided on	Width of Spectrogram	Where Measured	Temp.
May 12.403	31^m	$1^h\ 2^m$ E	13	*Fe* 30^smd	Star	0.160 R	Lower edge	+6°0+5°7 C.
13.388	29	1 38 E	13	30	Spectrum	.160	Center	+7°0
13.414	37	1 2 E	13	30	Spectrum	.634	Center	+7.0
17.422	46	0 47 E	13	33	Spectrum	.360	Center	+8.2
17.498	37	1 16 W	12	30	Star	.360	Lower edge	+8.0
20.413	38	0 47 E	13	30	Star	.160	" "	+7.7
20.438	34	0 2 W	12.5	30	Spectrum	.160	" "	+7.5
25.409	30	0 21 E	12.5	33	Spectrum	.160	Center	+6.0
25.461	47	0 54 W	12.5	30	Star	.160	Lower edge	
25.400	32	1 36 W	12.5	30	Spectrum	.360	Center	+5.8
26.478	36	1 23 W	12	30	Star	.160	Lower edge	+9.0
26.504	31	2 0 W	12	30	Spectrum	.160	" "	+8.7
29.422	37	0 14 W	12	33	Spectrum	.160	" "	+7.5
29.452	44	0 57 W	12	35	Star	.160	" "	+7.5
30.423	27	0 21 W	12	34	Star	.160	" "	+6.0
30.450	42	0 57 W	12	34	Spectrum	.160	" "	+5.8
31.421	37	0 21 W	12	33	Star	.160	" "	+7.2
31.450	40	1 2 W	12	33	Spectrum	.160	" "	+7.0

The light from the *Fe* spark always passes through a ground glass disk.
The camera setting was 13.2 for each of the above plates.

TABLE III*b*

n	λ	Δλ	Vel.
May 12			
0.151R	4294.296	+0.092t.-m.	+6.42km
9.925	4315.151	.056	3.89
19.928	4337.236	.020	1.38
26.863	4353.006	.075	5.16
48.590	4404.992	.065	4.43
57.501	4427.514	.094	6.37
63.297	4442.560	.050	3.37
69.647	4459.418	.061	4.10
94.331	4528.873	.075	4.97
96.749	4536.032	.040	2.65
		Mean	+ 4.27km
		Va	−11.86
		Vd	+ 0.06
		C.........	− 0.18
		ρ[1]	− 7.71
May 13 (1)			
0.217R	4294.434	+0.133t.-m.	+ 9.27km
9.987	4315.286	.077	5.35
14.932	4326.108	.149	10.33
19.987	4337.369	.153	10.58
26.904	4353.101	.057	3.93
34.788	4371.502	.060	4.12
38.142	4379.490	.094	6.43
46.900	4400.802	.064	4.36
48.595	4405.004	.077	5.24
50.071	4408.685	.103	7.00
52.746	4415.407	.114	7.74
57.511	4427.540	.058	3.23
63.321	4442.624	.114	7.69
69.649	4459.423	.122	8.20
75.875	4476.340	.085	5.69
82.533	4494.867	.129	8.61
94.368	4528.982	.053	3.51
96.794	4536.167	.073	4.82
102.324	4552.794	.069	4.54
106.602	4565.202	.060	3.94
		Mean.....	+ 6.26km
		Va	−12.24
		Vd........	+ .08
		C.........	− .18
		ρ..........	− 6.08
May 13 (2)			
0.230R	4294.461	+0.160t.-m.	+11.18k.m.
9.993	4315.289	.151	10.49
20.000	4337.398	.182	12.58
23.266	4344.779	.109	7.52
26.919	4353.134	.090	6.20
36.800	4376.283	.176	12.06
38.171	4379.560	.164	11.03
48.611	4405.044	.117	7.97
49.385	4406.972	.162	11.03
49.805	4408.020	.149	10.14
57.530	4427.588	.106	7.18
69.672	4459.485	.128	8.61
78.130	4482.565	.127	8.50
94.887	4529.047	.118	7.81
118.474	4603.444	.163	10.62
May 13 (2)—*Continued*			
		Mean.....	+ 9.54km
		Va........	−12.25
		Vd........	+ .06
		C.........	− 2.38
		ρ..........	−5.03
May 17 (1)			
9.652R	4314.559	+0.129t.-m.	+ 8.96km
10.000	4315.314	.105	7.30
19.995	4337.387	.171	11.82
23.268	4344.783	.113	7.80
24.768	4348.201	.071	4.89
26.917	4353.130	.086	5.93
36.804	4376.293	.186	12.75
49.380	4406.959	.149	10.14
57.525	4427.573	.091	6.17
63.346	4442.689	.179	12.08
		Mean.....	+ 9.37km
		Va........	−13.72
		Vd........	+ .05
		C.........	− .76
		ρ..........	− 5.06
May 17 (2)			
0.158R	4294.301	+0.106t.-m.	+ 7.40km
9.939	4315.160	.022	1.53
26.867	4353.015	.107	7.37
34.768	4371.456	.088	6.04
48.575	4404.955	.028	1.91
50.060	4408.658	.076	5.17
57.498	4427.506	.086	5.83
72.409	4466.874	.147	10.95
75.844	4476.255	.070	4.69
94.344	4528.911	.113	7.48
96.756	4536.053	.174	11.50
111.668	4581.710	.135	8.84
		Mean.....	+ 6.56km
		Va........	−13.75
		Vd........	− .07
		C.........	− .76
		ρ..........	− 8.02
May 20 (1)			
0.155	4294.304	+0.100t.-m.	+ 6.98km
9.944	4315.193	.098	6.81
14.891	4326.020	.081	5.62
26.852	4352.980	.072	4.96
34.761	4371.440	.072	4.94
50.052	4408.637	.055	3.74
57.501	4427.514	.094	6.37
69.639	4459.397	.096	6.46
73.433	4469.659	.114	7.65
82.499	4494.775	.119	7.94
94.325	4528.855	.057	3.77
96.762	4536.071	.049	3.24
101.380	4549.931	.124	8.11

[1] The symbol ρ is used in these tables for the radial velocity of the star referred to the Sun.

TABLE III*b*—*Continued*

n	λ	$\Delta\lambda$	Vel.
May 20 (1)—*Continued*			
		Mean.....	+ 5.89km
		Va........	−14.76
		Vd........	− .05
		C.........	− .18
		ρ	− 9.10
May 20 (2)			
9.996R	4315.306	+0.168t.-m.	+11.68km
14.945	4326.139	.150	10.40
19.008	4335.173	.139	9.61
26.513	4352.201	.118	8.13
26.928	4353.155	.111	7.65
29.854	4359.927	.143	9.83
50.002	4408.737	.155	10.54
57.537	4427.607	.125	8.46
69.664	4459.463	.162	10.89
78.122	4482.543	.105	7.03
94.377	4529.000	.080	5.30
96.802	4536.189	.095	6.28
		Mean.....	+ 8.82km
		Va........	−14.76
		Vd........	.00
		C.........	− .18
		ρ.........	− 6.12
May 25 (1)			
10.006R	4315.327	+0.118t.-m.	+ 8.19km
14.621	4325.424	.118	8.17
19.019	4335.220	.136	12.86
26.936	4353.174	.130	8.96
36.802	4376.388	.181	12.41
49.383	4406.966	.156	10.61
49.809	4408.040	.169	11.50
57.538	4427.609	.189	12.80
65.380	4448.047	.155	10.45
69.678	4459.501	.144	9.64
94.387	4529.038	.109	7.21
96.807	4536.205	.157	10.38
		Mean.....	+10.27
		Va........	−16.41
		Vd........	+ .02
		C.........	− .18
		ρ..........	− 6.30
May 25 (2)			
0.186R	4294.369	+0.096t.-m.	+ 6.70km
9.964	4315.236	.098	6.81
26.892	4353.073	.142	9.78
29.825	4359.850	.075	5.13
34.756	4371.428	.060	4.12
36.740	4376.141	.034	2.33
50.058	4408.662	.080	5.44
57.509	4427.534	.114	7.72
69.638	4459.396	.095	6.39
72.417	4466.896	.133	8.93
78.088	4482.438	.100	6.69
94.335	4528.974	.163	10.79
May 25 (2)—*Continued*			
96.767	4536.086	.177	11.70
101.386	4549.949	.141	9.29
		Mean.....	+ 7.27km
		Va.......	−16.45
		Vd........	− .06
		C.........	− .18
		ρ..........	− 9.42
May 25 (3)			
0.235	4294.472	+0.171t.-m.	+11.94km
10.012	4315.340	.131	9.10
14.639	4325.464	.163	11.30
23.279	4344.808	.138	9.52
26.932	4353.164	.233	16.05
29.879	4359.985	.176	12.11
50.103	4408.765	.143	9.72
57.540	4427.614	.194	13.14
69.693	4450.538	.181	12.17
78.153	4482.628	.190	12.72
83.124	4496.538	.220	14.67
94.390	4529.047	.236	15.63
96.823	4536.253	.205	13.74
106.636	4566.008	.166	10.90
		Mean.....	+12.34km
		Va........	−16.46
		Vd........	− .08
		C.........	− .76
		ρ..........	− 4.96
May 26 (1)			
0.184	4294.365	+0.161t.-m.	+12.24km
9.978	4315.266	.128	8.89
26.896	4353.082	.174	11.09
49.778	4407.953	.143	9.73
57.513	4427.545	.125	8.47
69.649	4459.423	.122	8.20
75.867	4476.312	.127	8.51
94.342	4528.906	.108	7.15
96.778	4536.119	.097	6.41
		Mean.....	+ 9.07km
		Va........	−16.78
		Vd........	− .07
		C.........	− .18
		ρ..........	− 7.96
May 26 (2)			
0.205	4294.409	+0.136t.-m.	+ 9.50km
9.979	4315.266	.128	8.89
14.602	4325.382	.199	13.80
19.986	4337.367	.151	10.42
21.103	4338.882	.151	10.43
26.920	4353.137	.093	6.41
57.515	4427.550	.130	8.80
65.389	4448.070	.178	12.00
69.655	4459.439	.138	9.28
78.121	4482.540	.165	11.04
83.108	4496.493	.175	11.67

TABLE III*b*—*Continued*

n	λ	Δλ	Vel.
May 26 (2)—*Continued*			
94.372	4528.995	.197	13.04
101.431	4550.085	.185	12.20
113.213	4586.593	.185	12.10
		Mean.....	+10.68km
		Va........	−16.78
		Vd........	− .10
		C.........	− .18
		ρ..........	− 6.38
May 29 (1)			
10.005R	4315.325	+0.187t.-m.	+12.99km
14.953	4326.156	.197	13.66
20.012	4337.425	.209	14.45
26.931	4353.162	.118	8.13
36.810	4376.307	.200	13.71
52.779	4415.489	.196	13.31
57.539	4427.612	.130	8.80
63.363	4442.734	.208	14.04
72.463	4467.020	.134	8.99
75.891	4476.384	.199	13.34
94.381	4529.021	.223	14.76
96.814	4536.226	.132	8.73
		Mean.....	+12.08km
		Va........	−17.68
		Vd........	− .01
		C.........	− .18
		ρ..........	− 5.79
May 29 (2)			
0.169R	4294.333	+0.129t.-m.	+ 9.01km
9.951	4315.208	.113	7.85
26.888	4353.064	.133	9.16
29.837	4359.887	.103	8.08
34.781	4371.487	.119	8.16
50.069	4408.680	.098	6.66
57.518	4427.557	.137	9.28
69.653	4459.434	.133	8.94
78.092	4482.459	.121	8.10
94.344	4528.011	.113	7.48
96.793	4536.160	.112	7.41
		Mean.....	+ 8.19km
		Va........	−17.69
		Vd........	− .06
		C.........	− .18
		ρ..........	− 9.74
May 30 (1)			
0.176R	4294.348	+0.144t.-m.	+10.05km
9.960	4315.227	0.089	6.19
14.998	4326.256	.137	9.50
21.067	4339.800	.183	12.64
26.470	4352.102	.172	11.85
36.742	4376.145	(.038).	(2.60)
50.079	4408.705	.123	8.36
57.512	4427.549	.122	8.27
69.662	4459.458	.157	10.56
94.346	4528.917	.119	7.88
96.773	4536.104	.195	12.89
May 30 (1)—*Continued*			
		Mean.....	+ 9.89km
		Va........	−17.99
		Vd........	− .02
		C.........	− .18
		ρ..........	− 8.37
May 30 (2)			
10.005R	4315.325	+0.116t.-m.	+ 8.06km
14.635	4325.446	.141	9.71
19.030	4335.007	.174	12.03
26.935	4353.172	.128	8.82
36.818	4376.326	.219	15.01
50.108	4408.777	.155	10.54
52.785	4415.504	.211	14.87
57.564	4427.676	.194	13.14
69.708	4459.591	.224	15.03
78.158	4482.642	.204	13.65
94.403	4529.086	.157	10.40
		Mean.....	+12.01km
		Va........	−17.99
		Vd........	− .06
		C.........	− .18
		ρ..........	− 5.32
May 31 (1)			
9.604R	4314.455	+0.174t.-m.	+12.09km
21.465	4340.699	(.065)	(+ 4.49)
26.892	4353.073	.142	9.78
50.069	4408.680	.098	6.67
52.742	4415.395	.102	6.93
57.508	4427.532	.112	7.59
66.364	4450.655	.173	11.64
69.653	4459.434	.134	9.01
75.863	4476.307	.122	8.18
83.075	4496.400	.179	11.94
96.764	4536.007	.168	11.11
		Mean.....	+ 9.23km
		Va........	−18.27
		Vd........	− .01
		C.........	− .18
		ρ..........	− 9.23
May 31 (2)			
6.715R	4308.225	+0.202t.-m.	+14.06km
9.975	4315.260	.166	11.53
14.938	4326.123	.184	12.76
26.932	4353.164	.233	16.05
38.179	4379.579	.183	12.53
49.110	4406.287	.180	12.26
50.110	4408.782	.160	10.88
57.540	4427.614	.194	14.74
69.674	4459.490	.189	12.71
94.377	4529.009	.211	13.96
96.804	4536.196	.204	13.49
		Mean.....	+13.27km
		Va........	−18.28
		Vd........	− .06
		C.........	− .18
		ρ..........	− 5.25

Values enclosed in parentheses were not used.

TABLE IV

SUMMARY

	Lower Edge	Red. to Center		Lower Edge	Red. to Center	
May 12	−7.39 km	+2.85 km	May 13 (1)	−6.08 km	0.00 km	−6.08 km
17 (2)	8.02	2.85	13 (2)	5.03	0.00	5.03
20 (1)	9.10	2.85	17 (1)	5.06	0.00	5.06
25 (2)	9.42	2.85	20 (2)	5.95	+1.02	4.93
26 (1)	7.96	2.85	25 (1)	6.30	0.72	5.58
29 (2)	9.74	2.85	25 (3)	4.96	0.72	4.24
30 (1)	8.37	2.85	26 (2)	6.38	1.11	5.27
31 (1)	9.23	2.85	29 (1)	5.79	0.77	5.02
			30 (2)	5.32	0.52	4.80
Mean	−8.65		31 (2)	5.25	0.53	4.72
	+2.85					
	−5.80				Mean	−5.07 km

Final mean, radial velocity = −5.44 km at epoch 1904.39.

Table IV gives a summary of the velocities thus obtained. We see that for the plates obtained when the visual star was held on the slit the velocity differs 0.5 km from that given when the spectrum was kept at its brightest by guiding. If we collect the observations of this star, we obtain the following values:

Potsdam, 1889, - - - - - -	−7.7 km
Keeler, - - - - - - -	6.9
Bélopolsky, 1893, - - - - - -	5.7
Frost and Adams, 1902, - - -	4.3
Newall, 1903, - - - - - -	5.8
Frost and Adams, 1903, - - -	4.8
Bélopolsky, 1903, - - - - - -	6.1
Bélopolsky, 1904, - - - - -	5.5
Mean, - - - - - - - -	−5.85

But if we combine only the results since 1902, we obtain −5.3 km. We might, therefore, infer that guiding on the brightest part of the spectrum yielded a result almost free from error, but by guiding with the visual star upon the slit we obtain a spectrogram which, on account of the diffuseness of the lines and their inclination to the artificial lines, give worse results. In this case it is especially important in making the settings under the microscope that the center of the spectrum is measured.

It thus appears that for the Poulkova instrument the false inclination of the stellar lines with respect to the comparison lines is due to

the fact that the photographic rays fall upon the collimator lens obliquely, and thus have a different path through the prisms from that of the rays from the iron arc. Hence a photographic plate placed just back of the collimator lens is illuminated over only half of the aperture.

I cannot yet decide to what extent this error enters in the case of spectrograms of planets. It is true that the lines in the spectrum of the Moon show no inclination with respect to the comparison lines. But the fact that in the case of the stellar spectra the inclination changes its sign when the spectrograph is rotated 180° about the optical axis of the thirty-inch refractor, would indicate that in the case of the planets the rotation of the spectrograph does not furnish a criterion of the true inclination. *Jupiter* is also not suitable for serving as a check, as the great variations of the velocity by zones requires that the setting should be made on precisely the same points of the disk, which is exceedingly difficult if the magnification of the guiding telescope is insufficient. In our case, the further circumstance enters that the visual disk does not coincide with the photographic.

TABLE V

ROWLAND'S WAVE-LENGTHS OF LINES USED

Rowland	Int.	Blend	Rowland	Int.	Blend
λ		λ	λ		λ
4294.204	2	4294.273	4459.199	2	4459.260
4.310	5		9.301	3	
					4459.357
4307.907	3	4308.023	4459.525	1	
8.081	6				
			4466.727	5	
4314.248	3	4314.281			
4.381	1		4476.185	4	4476.214
4.479	1	4314.430	6.253	3	
4314.964	1	4315.095	4482.338	5	4482.376
5.138	3		2.438	3	
5.262	4	4315.209			
			4494.738	6	
4325.152	4	4325.183			
5.306	1		4496.125	1	
			6.318	1	
4325.939	8	4325.959			
6.119	1		4528.798	8	4528.811
			8.929	0	

TABLE V—*Continued*

Rowland	Int.	Blend	Rowland	Int.	Blend
4334.965	0	4335.034			
5.102	0		4535.879	1	4535.992
			5.909	0	
4337.216	5		6.094	2	4536.022
					4536.048
4339.617	4	4339.731	4549.808	6	4549.814
9.882	3		9.990	0	
					4559.814
4340.634	20		4559.802	6	
			9.900	0	4565.854
4344.670	4				
			4565.842	2	
4348.130	1	4348.045	5.905	00	
8.003	2				
			4581.575	4	4581.635
4351.930	5	4352.007	1.694	4	
2.083	5				
			4586.408	1	
4352.908	4	4352.931	6.552	1	
3.044	0				
			4603.126	6	
4359.784	3	4359.809			
9.907	0				
4371.221	1	4371.368			
1.442	2				
4376.107	2				
4379.396	4				
4400.738	1	4400.601			
0.551	3				
4404.927	10				
4406.810	2				
4407.810	2	4407.851			
7.871	4				
4408.583	3	4408.622			
8.683	2				
4415.293	8				
4427.266	2	4427.420			
7.482	5				
4442.510	6	4442.526			
2.621	1				
4447.892	6				
4450.482	1	4450.597			
0.654	2				

Table V gives the wave-lengths from Rowland's table which I have used. The column "Blend" contains the wave-lengths of the lines which lie so close together that they do not appear separated on the stellar spectrogram. In such cases the weight was assigned proportionately to the intensity.

POULKOVA, November 1904.

ON THE SPECTRUM OF MAGNESIUM

By JAMES BARNES

The magnesium spectrum is of much interest on account of its presence in the spectrum of many[1] stars, and also on account of its application in the determination of stellar[2] temperatures.

The line λ 4481 appears very strong in the spectra of numerous stars belonging to Vogel's first type, while λ 4352 is very faint or not present. From these facts Scheiner drew the conclusion that on the stars of the first type the temperature of the absorbing layer was approximately that of the electric spark, while on the stars in whose spectrum the line λ 4352 more strongly occurs the temperature was about that of the electric arc. Just what Scheiner meant by the temperature of the spark is not clear, for the words "temperature of the spark" have in themselves no meaning according to our present ideas based on the kinetic theory of gases. Recently Hartmann[3] has shown that the presence or absence of these lines is no indication of high or low temperatures; that is, the lines λ 4481 and λ 4352 are not due to temperature, but rather to electrical causes.

The so-called spark lines are those which appear in the electric spark produced by a high-tension discharge and are rarely present in the arc running under the ordinary voltage and current. The arc lines are those that appear in the arc and are of weak intensity or not visible in the spark discharge.

It was the object of this work to repeat the observations of Hartmann, and also to study the other conditions which might be found to enhance or diminish the intensity of these lines.

I need only briefly refer to the work of Liveing and Dewar,[4] who first observed the presence of spark lines in the arc produced between thick electrodes of magnesium, when surrounded by air, carbonic acid, ammonia, etc. From their results they threw doubt on the then accepted opinion that the temperature of the spark discharge

[1] H. C. Vogel, *Astronomische Nachrichten*, **161**, 365, 1903.

[2] H. Kayser, *ibid.*, **162**, 277, 1903.

[3] *Astrophysical Journal*, **17**, 270, 1903.

[4] *Proc. R. S.* **44**, 241, 1888.

was much higher than that of the arc. They believed that the production of the spark lines was conditioned by the energy of the electric discharge, and not by any change in the temperature.

This important result led the way for further investigation. Crew[1] found the spark line λ 4481 to be one of the strongest lines in the spectrum obtained with his "rotating" arc. This line was intensified when the arc was surrounded by hydrogen, while the lines belonging to the Kayser and Runge series were unaffected. Porter[2] with the same arc in nitrogen found this line reduced to about one-fifth of its intensity in air. Schenck[3] observed that the intensity of the line λ 4481 decreases in the spark spectrum if the electrodes are heated to the point of melting. Hartmann[4] and Eberhard observed that the spark line appeared in the spectrum of the arc under water, the effect being analogous to that produced by hydrogen on the arc. Recently Hartmann[5] has been able in the case of magnesium and bismuth to transform the arc spectrum into the spark spectrum without changing in any way the surrounding dielectric, by merely diminishing the strength of the current. From the results of this very careful experiment he has conclusively shown, as was believed by Liveing and Dewar, that the presence of the line λ 4481 is no proof of high temperatures, but is rather due to electro-luminescence.

Hartmann concludes, however, that λ 4481 results from the vibrations of particles highly charged with electricity, and that the charge carried by a particle is a function of the resistance of the arc. The greater the conductivity, the less the charge, and hence a smaller intensity of this line. This conclusion is partly based on the observation that the intensity of λ 4481 decreases if the dielectric surrounding a spark is reduced by exhaustion. In the arc he considers that the same effect happens. This explanation was not satisfactory to me, and was overthrown when it was found that experiments gave just the opposite result, namely, that the line λ 4481 was enhanced when the density of the dielectric surrounding the arc was diminished.

The work was carried on with the following apparatus: A Rowland concave grating of about thirteen feet radius was used, and photo-

[1] *Phil. Mag.*, (5) **38**, 379, 1894.

[2] *Astrophysical Journal*, **15**, 274, 1902.

[3] *Ibid.*, **14**, 116, 1901.

[4] *Ibid.*, **17**, 229, 1903.

[5] *Ibid.*, **17**, 270, 1903.

graphs were taken in the first spectrum, since in this order it was the most brilliant. Later in the work a smaller grating with radius of 60 cm was found sufficient for the problems in view.

Solid magnesium rods of about 1 cm diameter were employed as electrodes. These, suitably mounted, were inclosed in a glass vessel of 800 cu.cm capacity and having a long neck, over the end of which was sealed a piece of plate glass. In the two openings in the sides of the vessel were placed rubber stoppers conveying the supports for the electrodes and the electrical connections. A glass tube connects with the hydrogen generator and the Geryk exhaust pump. Soon after the arc is started, the bulb of the vessel is covered with a deposit of magnesium thrown off from the arc; this, however, does not penetrate into the neck, so that the radiation passes through to the slit of the grating without loss. The rubber stoppers allowed the electrodes to be slightly moved without affecting the pressure in the vessel. In this way the electrodes could be brought together to strike the arc. The hydrogen was obtained by the ordinary method of the action of hydrochloric acid upon granulated zinc. It was passed through sulphuric acid before entering the vessel containing the arc.

The current was obtained from a 110-volt circuit, and its strength was varied by resistances consisting of incandescent lamps. When the spark was employed it was produced by an ordinary induction coil.

With the magnesium arc burning in air the electrodes soon became coated with the oxide, and this had to be removed when small current-strengths were employed before an arc could be again started. As the arc had to be made a great number of times during one exposure, plates were taken only when the air surrounding the arc was at atmospheric pressure, when the magnesium oxide could be easily removed; and also at very low pressures, when the little oxide formed did not interfere with the striking of the arc whenever required. At other pressures below the atmospheric it was impossible to clean the electrodes without opening the vessel and thereby changing the pressure. As hydrogen does not unite with magnesium, observations could be obtained without difficulty at any pressure.

RESULTS

After taking a systematic series of photographs of the spectrum of the arc under different current-strengths and pressures of the surrounding gas, it was found that the intensities of the lines in the first subordinate series λλ 3838.4, 3832.4, and 3829.5, and in the second subordinate series λλ 5183.8, 5172.8, and 5167.5, were practically unaffected. These lines in the spark are also unaffected by any change in pressure of the surrounding dielectric. Thus the intensity of the line λ 5183.8 has been taken as the standard, which is called 10; a line barely visible on the negatives is given the intensity 1. The following tables contain the results obtained; the pressures are given in millimeters of mercury and the current-strengths in amperes.

ARC IN AIR

Wave-Length	Pressure	Intensity when Current-Strength is				
		7.5	3.5	2.0	1.0	0.5 amperes
4703.2 (arc line)....	760mm	10	10		10	8
	1		8	8	8	7
4571.3 (arc)........	760	6	6		4	4
	1		1	0	0	0
4481 (spark)........	760	0	1		10	15
	1		20	20	20	20
4352.1 (arc)........	760	10	10		10	8
	1		3	2	1	1
4167.8 (arc)........	760	7	7		7	5
	1		1	1	1	1
4058.4 (arc)........	760	5	5		5	4
	1		1	1	0	0

ARC IN HYDROGEN

Wave-Length	Pressure	Intensity when Current-Strength is	
		3.5	0.5 amperes
4703.2...........	760mm	6	2
	380	6	1
4481.............	760	5	15
	380	10	15
4352.1...........	760	4	0
	380	3	0

The other arc lines are omitted from the hydrogen part of the

table, as they do not appear on the plates at any pressure or current-strength.

These tables show that all the arc lines are weakened when the strength of the current is diminished, both at atmospheric and lower pressures when the dielectric is air or hydrogen. In air the change in intensity is slight. Also, with the same current and pressure hydrogen diminishes the intensity of the arc lines, while the spark line is enhanced, which is in accordance with the results of Crew. It is also seen how the remarkable spark line λ 4481 is intensified in air as the current is decreased, thus confirming the observations of Hartmann. In both air and hydrogen, using the arc, a decrease of the pressure always weakens the arc lines and intensifies the spark lines. It is important to note that the intensity of the line λ 4481 in a vacuum of about 1 mm pressure does not change its intensity with the variation of current throughout the range employed. It is also worth remarking that the plate taken of the spectrum of the spark obtained with an induction coil having a Leyden jar in parallel with the secondary was so similar to the one taken of the spectrum of the arc in a vacuum that almost no difference was perceptible. The distance between the electrodes in all the observations was not more, generally less, than a few millimeters. The voltage across the electrodes was about forty volts.

With regard to the spectrum of zinc, cadmium, and bismuth very few conclusive results were obtained. If a transformation of the arc spectrum into the spark spectrum happened, it was much less striking than in the case of magnesium, where the steps were easily obtained. The observation of Hartmann that the zinc spark lines λ 4912 and λ 4925 appeared in the arc spectrum on reducing the current to 0.5 ampere was corroborated. In a vacuum it was found that these spark lines made their appearance in the arc when not a trace of them could be found in the arc surrounded by air at atmospheric pressure, using the same strength of current. In the case of cadmium and bismuth I was unable to satisfy myself of the appearance of the spark lines in the arc in a vacuum.

These results may be summarized as follows:

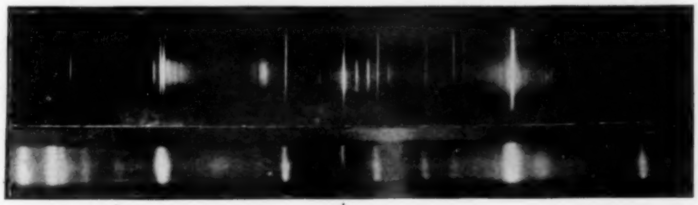
Magnesium.—In the arc in air at atmospheric pressure the arc lines are weakened as the strength of the current is diminished,

while the spark line λ 4481 is increased in intensity. For all strengths of current the arc lines are weakened as the pressure is decreased, and just the reverse is the case with the spark line. The same phenomena occur in hydrogen, but to a more marked degree.

The line λ 4481, when obtained with an arc in a vacuum, does not change its intensity with any variation in the current-strength.

Zinc.—In the arc in air the spark lines λ 4912 and λ 4925 made their appearance when the current was diminished. These lines also appeared very clearly in the arc in a vacuum, while they were absent in the arc in air using the same current-strength.

Since the above observations were completed, an important paper by Professor Crew appeared in the *Astrophysical Journal* for Novem-



1. Spectrum of Magnesium Arc in a vacuum (Current 3 amperes).
2. Showing λ 4481 at cathode.

ber 1904, in which he says that a high E.M.F., rapidly changing, is the essential condition for the production of spark lines in arc spectra. The above results can all be explained by this proposition. For just as hydrogen introduces a greater electromotive force on making and breaking the arc than in air, so also we might expect that the diminution of the density of the dielectric might have a similar effect.

An observation was made which bears directly on the cause of the line λ 4481, and is that it appears principally in the portion of the arc near the negative electrode, as is shown in the spectrum of Fig. 2. This photograph, as well as a number of others giving the same result, was obtained by removing the slit of the spectroscope and placing the arc in its place. For an exposure the arc was only made a second or less at atmospheric pressure with a current of 3 amperes. This result adds further evidence for Crew's suggestion

that spark lines are associated with steep potential gradients, since we know there is a rapid fall of potential at each electrode in the arc.

At atmospheric pressure the free path of an ion is so small that the necessary acceleration for spark lines is only produced close to the electrodes, but as the density of the gas or vapor is diminished, the free paths of the ions becoming longer, one would naturally expect to find the characteristic radiation farther out, and with sufficient exhaustion may reach the entire distance between the electrodes, as is the case in the above results at the pressure of 1 mm. On the same assumption, Hartmann's results as to the increase of intensity of the spark line with decrease of current-strength can be explained, for with a greater current more of the metal electrodes is volatilized, producing an increase of the density.

In the ordinary arc the anode is the hotter, and hence there is a greater density of the vapor in its vicinity. Hence the above hypothesis would account for the greater strength of λ 4481 at the cathode over that at the anode.

In conclusion the author wishes to thank Professor Ames for many suggestions made during the work.

Physical Laboratory,
Johns Hopkins University,
December 1904

Minor Contributions and Notes.

GRANT BY THE SMITHSONIAN INSTITUTION TO THE ASTROPHYSICAL JOURNAL

The editors of the *Astrophysical Journal* again have the pleasant duty of publicly acknowledging the receipt from Doctor S. P. Langley, Secretary of the Smithsonian Institution, of a check for $200 in continuance of his previous liberality. This exceedingly important financial assistance to the *Journal* makes it possible to supply fifty subscriptions to individuals in America and abroad who do not otherwise take the *Journal*, but who, it is hoped, value it.

NOTE ON ADDITIONAL TRIPLETS IN THE ARC SPECTRUM OF STRONTIUM

Photographs of the arc-spectrum of strontium recently obtained by the writer show two unmistakable triplets in the violet which do not appear to have been previously recorded, but which are of special interest on account of their series connection with two of the four narrow triplets to which attention has been drawn by Kayser and Runge. Particulars of the new triplets, and of those having equal frequency intervals already recognized by Kayser and Runge, are given in the accompanying table.

An inspection of the photographs at once suggested that the last four triplets in the list were members of a series of the usual type. The frequency intervals are nearly equal, the intensities gradually diminish as the violet is approached, and all the lines are shaded toward the red. Calculation establishes a series relationship. By taking values of $m=4$, 5, 6, the least refrangible members of the triplets beginning at λ 4892, 4338, and 4087 give the following numerical solution to Kayser and Runge's well-known series formula:

$$n=27646.6-\frac{114374.6}{m^2}-\frac{16160.8}{m^4}$$

Substituting $m=7$, the resulting value of n for the next member of the series is 25305.7, which, taking account of the nebulous character of the lines, is in good agreement with the observed value for the first line of the triplet beginning at λ 3951.

NARROW TRIPLETS IN THE STRONTIUM ARC SPECTRUM

	Wave-Length	Intensity and Character	Frequency *in vacuo*	Frequency Intervals
(K. & R.).	5535.01	6	18061.9	100.2
	5504.48	10	18162.1	59.9
	5486.37	8	18222.0	
	5257.12	10	19016.6	100.4
	5229.52	8	19117.0	59.7
	5213.23	4	19176.7	
	4892.20	8	20435.1	97.7
	4868.92	6 n	20532.8	57.7
	4855.27	6 n	20590.5	
	4338.00	6 b^{v}	23045.8	99.2
	4319.39	4 b^{v}	23145.0	58.6
	4308.49	2 b^{v}	23203.6	
(Fowler)	4087.67	3 b^{v}	24457.1	100.1
	4071.01	2 b^{v}	24557.2	59.2
	4061.21	2 b^{v}	24616.4	
	3950.96	2 b^{v}	25303.3	100.6
	3935.33	1 b^{v}	25403.9	58.6
	3926.27	1 b^{v}	25462.5	

n denotes nebulous; b^{v}, that the line was nebulous on the side toward the red.

Uniting all four triplets in the formula discussed by Mr. Shaw and the writer,[1] the resulting equation for the less refrangible components is

$$n=27603.5-\frac{108065.6}{(m+1.817677)^2+0.50064},$$

in which m has the values 2, 3, 4, 5, for the four lines observed.

Taking $m=3$ in the Kayser and Runge formula, another triplet would be expected with its least refrangible member in the neighborhood of λ 6783, while the second formula, with $m=1$, would predict it near λ 6755. Careful observations in this region, however, have failed to reveal any such triplet, and it would seem that, besides other peculiarities, strontium fails to show this triplet, or has it very feebly developed. In this respect strontium seems to resemble potassium, in which the first doublet of the second subordinate series is so feeble that it has only lately been detected.[2]

A. FOWLER.

ROYAL COLLEGE OF SCIENCE, LONDON.
November 1904.

[1] *Astrophysical Journal*, **18**, 21, 1903.   [2] *Ibid.*, **20**, 196, 1904.